Some Constraints on Possible Worlds in Semantics and Pragmatics

by

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ABSTRACT

This dissertation explores some ways that possible worlds are constrained in formal semantics and pragmatics. While the first two chapters form a unified investigation of a particular intensional construction, the third addresses pragmatic constraints on discourse.

The two first chapters focus on a certain reading of intensional sentences—*the fourth reading*—in which a DP appears to scope above an intensional operator which in turn determines the evaluation domain of its restrictor. It is a near-consensus in the semantic literature that this reading is generally unattested, and mainstream theories of intensionality are designed to block its generation. Szabó (2010) offers a dissenting view, providing examples of the reading involving a range of intensional operators. In the first chapter, I explore Szabó's examples involving modals—i.e., quantifiers over possible worlds—and show that they are restricted to *relativization* environments and can be derived from features of relativization without challenging mainstream theories of intensionality. However, the following chapter then shows that once these theories are paired with standard mechanisms that derive *cumulative inferences*, they actually derive a range of unattested modal fourth readings. This finding supports recent work by Schmitt (2023) arguing that possible worlds never interact with cumulation mechanisms. I show that in that sense, worlds are set apart from time intervals, as the fourth reading is attested in the temporal domain and should be derived via cumulation.

The final chapter, co-authored with Omri Doron, shifts to pragmatics, arguing that discourse is governed by a constraint requiring that declarative sentences in discourse not provide information that is unsolicited given the *Question under Discussion (QuD)* relative to which they were uttered; i.e., that they not distinguish among possible worlds within the same cell in the partition imposed by the QuD. We defend this constraint against undergeneration challenges and introduce diagnostics showing that the constraint is active in discourse.

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Yael, Zohara, and Ivri,
I love you.

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Chapter 1

Introduction

This thesis is a collection of three papers. Two are quite intimately connected, and have to do with what has been called in the semantic literature "*the fourth reading*" or "*the wide-scope opaque reading*" of DPs in intensional contexts. Since they have to do with the same subject matter, there is some repetition of expository materials in these chapters, so that they could be read as stand-alone papers. The final chapter, co-authored with Omri Doron, concerns the pragmatic constraints that regulate how declarative sentences should address the questions relative to which they are uttered. This introduction provides short summaries of each of the chapters.

Chapter 2 It is a near-consensus view in the semantic literature that the interaction between quantificational DPs and intensional operators does not give rise to *wide-scope opaque* readings of intensional sentences; namely, readings in which a DP in a sentence outscopes an intensional operator while its restrictor is evaluated opaquely, in the operator's domain of quantification. In a dissent, Szabó (2010, 2011) presents a set of intensional sentences—cutting across the three primary intensional environments: attitudes, modals and tense—argued to exemplify exactly this reading, and deduces from the apparent generality of the data that an unrestricted *bare quantification* mechanism is in charge of accounting for it.

We argue that the phenomenon is not as general as claimed, illustrating that Szabó's wide-scope opaque reading arises for modal sentences only when the modal they contain is embedded in a relative clause, and the determiner under which the relative clause itself is embedded scopes over that modal, while the NP heading the relative clause is interpreted opaquely. We then argue that the problematic reading can thus be derived without non-standard scoping mechanisms, as the byproduct of *reconstruction* into intensional rela-

tive clauses. Since reconstruction effects are strictly constrained, our approach makes testable predictions regarding the distribution of Szabó's reading with modals, which are corroborated. While the chapter focuses exclusively on Szabó's examples with modals, it concludes by outlining the kind of approach we believe will prove to be most fruitful in accounting for examples involving attitude predicates, and chapter 3 then addresses Szabó's examples involving tense operators.

Chapter 3 Schmitt (2023) recently observed that worlds pattern differently from entities of all other types in environments that are sensitive to the *plural part-structure* of their sub-constituents: While pluralities of all types seem able to partake in *cumulative* relations, expressions previously argued to denote pluralities of worlds in particular seem unable to do so. Schmitt concludes that no natural language expressions denote world pluralities. This chapter develops a new argument for that conclusion, which challenges *ontological symmetry*: the view that worlds and times should receive parallel semantic representations.

The argument begins where the previous chapter left off; namely, with the generalization that *wide-scope opaque readings*—in which a DP takes wide scope over an intensional operator while its nominal restrictor is evaluated opaquely, in the domain of that operator—are unattested. For example, the sentence "*Two victims might be trapped under the rubble*" cannot mean that there are two individuals who might each be a trapped victim (but are not both victims). I apply the logic of Schmitt's argument, showing that standard theories of intensionality overgenerate this unattested reading when combined with independently motivated cumulation mechanisms. If predicates denote relations between individuals and worlds, cumulation can apply to these relations, deriving the unattested reading as a cumulative inference. Blocking this result requires a general ban on cumulation involving pluralities of worlds.

The temporal domain, I show—building on the observations in Szabó 2010, 2011—presents the opposite pattern; there, sentences do have a reading parallel to the one unattested in the modal domain. Semantic theory must therefore block cumulation over worlds while allowing it over times.

Chapter 4 This chapter concerns the constraints that govern the relationship between declarative sentences and the questions relative to which they are uttered in discourse. We distinguish positive constraints, which require an utterance to provide information that helps resolve a question, and negative constraints, which require an utterance to refrain from providing information

that was not solicited by the question—a distinction that dates back to the two clauses in Grice's (1975) *Maxim of Quantity*. We argue that discourse must be subject to a negative constraint, which requires that the meaning of a declarative sentence either be entailed by or contradict each cell in the *Question under Discussion (QuD)*. An obvious challenge to this approach is that a negative constraint potentially undergenerates many felicitous discourses, since speakers routinely produce answers that contain unsolicited information. We show, however, that this objection is methodologically problematic: Just about any proposed constraint on discourse—positive *or* negative—suffers from an apparent undergeneration problem and necessitates an auxiliary mechanism to capture the complexity of the empirical landscape. The standard mechanism usually appealed to, termed *QuD-accommodation*, lets an interlocutor address a question that is distinct from the one that they were explicitly asked. But then, felicity of an overt question-answer pair cannot be used as evidence against any particular constraint. Infelicity, by contrast, is harder to explain away, and provides more reliable evidence.

We offer two positive arguments for a negative constraint. First, only with a negative constraint does our pragmatic theory place an upper bound on the logical strength of answers. We show that this upper bound is needed to account for certain infelicitous over-informative answers. Second, we develop a diagnostic for QuD-accommodation, and show that it targets precisely those answers that violate the negative constraint, even when they are sub-optimal in other respects. Finally, we comment on how our argument interacts with other debates pertaining to the constraints that govern felicitous discourse.

Chapter 2

Reconstruction, bare quantification, and the fourth reading

2.1 Introduction

Fodor (1970) famously claimed that a DP in an intensional environment may vary along two independent dimensions: It can take *wide* or *narrow* quantificational scope relative to the intensional operator in the sentence, and it can have its NP restrictor evaluated *opaquely*, in the worlds quantified over by that operator, or *transparently*, in the matrix world of evaluation. This gives rise to a four-way ambiguity, summarized in Table 2.1: On its *de dicto* reading, a DP takes narrow quantificational scope relative to an intensional operator, and its restrictor is evaluated opaquely. On its *de re* reading, the DP takes wide quantificational scope and its restrictor is evaluated transparently. The *third* reading combines narrow quantificational scope with transparent evaluation, and the fourth reading, henceforth **R4**, combines wide quantificational scope with opaque evaluation.

Sentence (1) is claimed in Fodor 1970 to demonstrate this ambiguity. Its *de dicto* reading in (1a), is true as long as buying *any* inexpensive coat meets Mary's desires. Its *de re* reading in (1b) is verified by there being a *specific* inexpensive coat Mary wants to buy, but she need not know that it is inexpensive, or, in fact, even a coat. The *third* reading in (1c) is true if buying each of several objects would satisfy Mary's desires, and all happen to be inexpensive coats, again, possibly unbeknownst to Mary. Finally, the R4 is true if there is a particular object that in all worlds that satisfy Mary's desires

	Transparent	Opaque
Wide-scope (*specific*)	*de re*	*fourth reading?*
Narrow-scope (*non-specific*)	*third reading*	*de dicto*

Table 2.1: DPs in intensional contexts

is an inexpensive coat that she buys. On this approach, whether (1) asserts the existence of a specific coat depends on the determiner's scope relative to the attitude, whereas whether Mary is allowed to be ignorant about the fact that what she desires meets the description "*inexpensive coat*" depends on the restrictor's domain of evaluation.

(1) Mary wants to buy an inexpensive coat.

a. $\lambda w.\ \forall w' \in \text{DESIRE}_{Mary,w}\colon \exists x\colon \textit{cheap-coat}_{w'}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(de dicto)*

b. $\lambda w.\ \exists x : \textit{cheap-coat}_{w}(x) \wedge \forall w' \in \text{DESIRE}_{Mary,w}\colon \textit{buy}_{w'}(Mary, x)$ *(de re)*

c. $\lambda w.\ \forall w' \in \text{DESIRE}_{Mary,w} : \exists x\colon \textit{cheap-coat}_{w}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(third)*

d. $\lambda w.\ \exists x\colon \forall w' \in \text{DESIRE}_{Mary,w}\colon \textit{cheap-coat}_{w'}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(R4)*

The literature since Fodor 1970, casts doubt on the empirical claim that (1) has the four readings above; in particular, the availability of the alleged R4 has been contested (cf. Keshet 2008; von Fintel & Heim 2011; Keshet & Schwarz 2019). If this reading is available, the sentence should be verified by scenarios in which Mary wants to buy a particular object, which is an inexpensive coat in Mary's DESIRE-worlds, though not necessarily in actuality. Assuming that Mary's DESIRE-worlds are constrained by what Mary *believes* (cf. Heim 1992), (1) is verified by the existence of an object that is actually expensive, which Mary wants to buy, thinking that it is inexpensive. As Keshet (2008: 30) points out, this is what (2a) means, but is not a possible interpretation of (1), as indicated by the fact that once pronominal reference is used to force wide scope for the DP over the attitude verb, as in (2b), we can no longer coherently deny that the coat is inexpensive.

(2) a. There's a coat that Mary wants to buy. She thinks it is inexpensive. But really, it is quite expensive.

b. Mary wants to buy an inexpensive coat.
#But really, it is quite expensive.

The consensus that has taken shape since Fodor 1970 is thus that it is a mistake to assume that the two interpretative dimensions in Table 2.1—namely,

wide *vs* narrow scope and transparent *vs* opaque evaluation—are truly independent of each other. Rather, wide scope for a DP relative to an intensional operator should block evaluation of its restrictor in the quantificational domain of the operator. In fact, as will be discussed in detail below, the semantic formalisms standardly used in the intensionality literature often have the non-existence of R4 built into them as a feature, as they reserve the possibility of opaque evaluation only to elements that are in the c-command domain of an intensional operator. On such an approach, as long as scope is determined structurally, wide-scope and opacity involve conflicting requirements, ruling out the possibility of R4.

As is usually the case, of course, not everyone subscribes to the consensus view. In particular, Szabó (2010, 2011) revived Fodor's empirical claim that R4 relative to attitude predicates exists and expanded on it, by arguing that the reading is not restricted to attitude reports, providing examples involving modals and tense operators as well. On this basis, Szabó also defends an expansive version of the main theoretical conclusion that Fodor draws from her empirical claim; namely, that opacity and wide scope should in fact be divorced one from the other and that we must posit a general mechanism dedicated to the generation of LFs in which a determiner outscopes an intensional operator that in turn determines the evaluation domain of its restrictor.

In this chapter we scrutinize a particular example, given in (3), offered by Szabó as an illustration of R4 relative to a modal operator.

(3) a. **Context:** In Alex's district, judges are elected, not nominated. Two candidates are running to fill a single vacancy, neither of whom is or has ever been a judge. The winner will preside over a hearing in which Alex is a defendant. To make sure that her lawyer prepares for both possible outcomes, Alex says:

b. There are two judges we could face in this court.

c. There are two individuals in the actual worlds who could each be the judge we face in court.

(adapted from Szabó 2011: ex. 26)

Now, Szabó (2011) does not say much about this case, except that the truth of (3b) given the scenario in (3a) indicates that it has a "split reading" in which the determiner "*two*" and the NP "*judges*" are evaluated in different positions with respect to the modal "*could*". This, indeed, seems to be the case: Since only one person is stipulated to preside over Alex's court case in (3a), (3b) is false if the determiner scopes below the modal, together with its adjacent NP. And given that there are no *actual* judges involved in (3a), only

potential ones, it is also false if the NP is evaluated above the modal together with the determiner. The sentence is only true in its respective context if it can be paraphrased with (3c), where the plural indefinite determiner "*two*" takes scope, and distributes,[1] over the modal "*could*", while the NP "*judges*" is evaluated within the intensional environment. Hence, the DP seems to have R4 relative to the modal.

This case is important, as it constitutes a crucial part of the motivation for Szabó's claim that R4 is a general phenomenon, cutting across the three primary intensional contexts—attitudes, modals, and tense. This claim, in turn, is taken to motivate a theory of intensionality and scope that fully divorces quantificational force from intensional status for almost any DP. In fact, Szabó posits a novel scope splitting operation, which allows a determiner to quantify unrestrictedly over the entire domain, with its apparent NP *restrictor*—obviously a misnomer on this approach—making its semantic contribution beneath some lower intensional operator within the nuclear scope of the unrestricted determiner. The resulting structure is schematized in (4), and is referred to—following Szabó—as *bare quantification*.

(4) [... Determiner ... [... Intensional operator ... [... NP ...]]]

In this present work, we will agree with Szabó that (3b) does, indeed, have a structure in which the determiner and its adjacent NP are evaluated in different positions relative to the modal. What we will contest, however, is that this lends support to a general bare quantification strategy in natural language. The gist of the argument is as follows: First, we argue that Szabó's proposed bare quantification mechanism overgenerates unattested cases of R4, while at the same time actually *undergenerating* the observed R4 of (3b). These observations will serve to undermine the initial motivation for bare quantification.

We will then show that the availability of R4 for (3b) is contingent on its particular syntactic profile: The sentence contains a *relative clause* that embeds a modal operator, with the alleged R4 involving wide quantificational force for the determiner embedding the relative clause, and evaluation of the NP adjacent to it in the scope of the clause-internal modal. But, we will argue, if that is the case, no novel scope displacement strategy is required to derive the R4 of (3b) to begin with; relative clauses are known to exhibit *reconstruction effects* whereby the NP adjacent to the clause-embedding determiner is actually interpreted clause-internally.

Thus, when a relative clause contains a modal, reconstruction already mimics the effects of bare quantification, and we can reduce the R4 of (3b)—and

[1] More on the role of distributivity below.

R4 relative to modal operators more generally—to reconstruction. Since this approach takes the presence of a relative clause to be a precondition for R4 with modals, it avoids the overgeneration issues faced by bare quantification. And, since the availability of reconstruction into a relative clause is subject to stringent constraints, the approach will also be shown to further restrict the distribution of R4 in a way that correctly captures the empirical landscape; it is exactly those environments that rule out reconstruction into a relative clause, that also block R4 relative to modals.

From the perspective of the consensus view, this is a welcome result, as it reduces a challenging observation to familiar means. It is important, however, to note at the outset what this chapter is *not* intended to achieve; that is, a full account of *all* of Szabó's examples of R4 that does not appeal to bare quantification. There are apparent cases of R4 relative to attitude predicates and tense operators that lack the relativization structure required to achieve R4 with modals, and thus cannot reduce to reconstruction effects. And while we briefly present some of these cases, and speculate on their nature in the conclusion, we leave a comprehensive treatment for another occasion. Our goals here are rather limited to:

(i) undermining the viability of Szabó's bare quantification mechanism;

(ii) motivating an account of the fragment of his data involving modals by appealing to independently-posited syntactic structures and within the confines of standard Montagovian semantics (Montague, 1973);[2] and

(iii) briefly outlining the kind of approach we believe will be most fruitful in accounting for the remaining data, given the findings in this chapter.

The chapter is structured as follows: Section 2.2 presents Szabó's data, discusses its design, and highlights the challenge it poses to mainstream theories of intensionality. Section 2.3 spells out Szabó's bare quantification approach and the empirical challenges it faces, illustrating that the data used to exemplify R4 with modals is undergenerated by bare quantification, which also overgenerates unattested readings for related modal sentences. Section 2.4 then provides a reduction of R4 relative to modals to the reconstruction of NPs into intensional relative clauses. The predictions of this reductive approach are corroborated in Section 2.5, and a potential challenge is discussed in Section 2.6. Section 2.7 presents Szabó's remaining cases of R4, which are

[2]In this respect, our approach contrasts with Moltmann's (2019, 2020b) account of similar data within Fine's (2017) non-standard framework of *truth-maker semantics*. Engaging with this work is not our primary goal, but footnote 15 highlights a strength of our account relative to this alternative.

not amenable to the treatment we provide for the modal cases, and concludes by speculating on how they should be approached.

2.2 The fourth reading and the challenge that it poses

2.2.1 The main data point

As noted above, our focus in this chapter will be Szabó's example of R4 relative to a modal operator, henceforth **R4-M**. The example, repeated in (5) for convenience, involves a sentence that seems true in a scenario that validates *only* its R4-M, suggesting that it must have a structure from which that reading, paraphrased in (5c), can be derived.[3]

(5) a. **Context:** In Alex's district, judges are elected, not nominated. Two candidates are running to fill a single vacancy, neither of whom is or has ever been a judge. The winner will preside over a hearing in which Alex is a defendant. To make sure that her lawyer prepares for both possible outcomes, Alex says:

b. There are two judges we could face in this court.

c. There are two individuals in the actual worlds who could each be the judge we face in court.

(adapted from Szabó 2011: ex. 26)

Note that, if existential modals are taken to quantify existentially over worlds, Szabó's case of R4-M in (5), contains two existential quantifiers: the modal and a plural *indefinite* DP. However, two existential quantifiers are scopally-commutative; i.e., $[\exists x \exists y \phi(x, y)] \Leftrightarrow [\exists y \exists x \phi(x, y)]$ for any scope ϕ. Thus, if scopal-commutativity is assumed to apply when one of the existential quantifiers involved is an intensional operator, R4 of an existential DP relative to such an operator is predicted to be equivalent to its *de dicto* reading, and

[3] Francez (2018) observed a phenomenon that at least superficially resembles Szabó's data, exemplified by an existential statement like "there can be three winners in this race", which can convey that three individuals might win the race. Here, too, the restrictor and determiner of a DP, "*three winners*" seem to scope in different places relative to a modal. However, such readings are subject to idiosyncratic restrictions (in particular, it is only available when relational nouns are involved), suggesting that they do not form a natural class with Szabó's data. We thus set this data aside and refer the reader to Francez 2018 for an account and to Nouwen 2022 for a recent discussion.

it is at least *prima facie* surprising that the two readings are distinguishable above in the first place.

R4 is nevertheless distinguishable from *de dicto* in (5b), with the existential modal, presumably due to the presence of *distributivity*. The determiner in (5b), while existential, is also *plural* and therefore ambiguous between a reading on which it distributes over elements it dominates and one on which it does not. Distributivity introduces universal quantification over atoms of the pluralities witnessing the indefinite, and *that* quantification is not scopally-commutative with the existential intensional operator. Thus, without distributivity, R4 of, e.g., (5b) in (6b) is equivalent to its *de dicto* reading (6a), whereas its distributive reading (6c) is not.[4]

(6) a. $\lambda w.\ \exists w' \in \text{ACC}(w) : \exists_2 x : \mathit{judges}_{w'}(x) \wedge \mathit{face\text{-}in\text{-}court}_{w'}(x)$ (*de dicto*)

b. $\lambda w.\ \exists_2 x : \exists w' \in \text{ACC}(w) : \mathit{judges}_{w'}(x) \wedge \mathit{face\text{-}in\text{-}court}_{w'}(x)$ (*non-distributive R4*)

c. $\lambda w.\ \exists_2 x : \forall y \in \text{ATOMS}(x) : \exists w' \in \text{ACC}(w) : \mathit{judges}_{w'}(y) \wedge \mathit{face\text{-}in\text{-}court}_{w'}(y)$ (*distributive R4*)

(For any w, we take $\text{ACC}(w)$ to be the set of worlds accessible from w, and for any $n \in \mathbf{N}$, we use $\exists_n x$ as shorthand for $\exists x : |\text{ATOMS}(x)| = n$)

Thus, whenever an example below contains a plural indefinite, we focus exclusively on its distributive R4, inserting the silent operator DIST (7) at LF when this is required to make a truth-conditional difference. However, we discuss in Section 2.4 cases in which R4-M arises with determiners that are not indefinite at all.

(7) $[\![\text{DIST}]\!] = \lambda f_{\langle e,t\rangle} \lambda x_e.\ \forall y \in \text{ATOMS}(x) : f(y)$

[4] As noted by a reviewer, whether commutativity applies when a modal is involved is subject to some controversy in the study of modal logic, and depends on whether we accept the *Barcan formula*, $\diamond\exists x : \phi x \rightarrow \exists x : \diamond\phi x$ and its converse, $\exists x : \diamond\phi x \rightarrow \diamond\exists x : \phi x$ (Barcan, 1946), which are not universally accepted. Together, the formulas require that the domain of individuals remain constant as we move from the evaluation index to those indices that are accessible from it. This is trivially validated in the semantic formalism we take for granted, in which sentences are evaluated relative to a model that contains domains of worlds and individuals, modals are quantifiers over the former, and the latter does not change from world to world. Our choice of formalism can, of course, be questioned, but we think this debate is beyond the scope of the current chapter, and thus circumvent it by focusing exclusively on the distributive construal of Szabó's data.

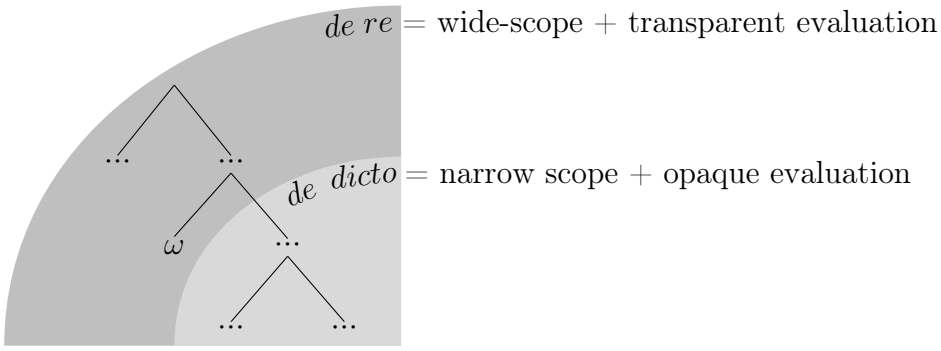


Figure 2.1: STI: wide-scope entails transparent evaluation; narrow-scope entails opaque evaluation

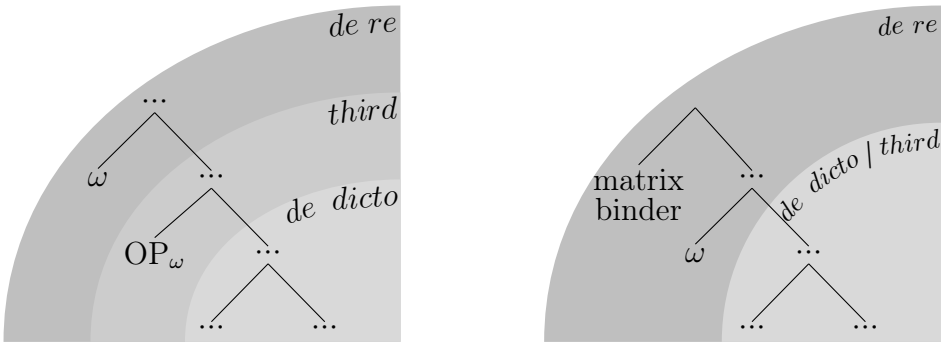
2.2.2 The problem

The existence of R4 poses a challenge for a prominent approach to intensionality, exemplified by the *modified scope theory* (Keshet 2008), and the *world-pronouns framework* (Percus 2000). Developed in response to the shortcomings of the classical *scope theory of intensionality* (henceforth, STI; Montague 1973; Ladusaw 1977 a.o.), both retain STI's supposedly innocuous features. However, exactly those features cause the undergeneration of R4. Our discussion below is an abstract review of some basic semantic theory, which may seem superfluous to an experienced reader, but is included here nonetheless to underscore the problem and the range of theoretical possibilities available in addressing it.[5]

STI maintains that every world-dependent element below an intensional operator ω at LF is evaluated in the worlds quantified over by ω. Paired with a syntactic approach to scope, STI predicts a biconditional dependence between quantificational scope and intensional status: as shown in Figure 2.1, any element under ω will receive narrow-scope and opaque evaluation relative to ω, and any element taking scope above ω will receive transparent evaluation. Biconditional dependence undergenerates not only R4, but also the *third* reading, which requires narrow-scope, but transparent evaluation.

Keshet (2008, 2010b) modifies STI by divorcing the quantificational aspect of ω from the mechanism determining the evaluation domain of material in its scope. While being below ω at LF entails narrow-scope, it is not sufficient for opacity. DPs are opaque only if they reside below *another operator*, OP_{ω}, itself situated below ω at LF. In the resulting system of *split intensionality*, there

[5]The discussion closely follows K&S 2019.



(a) Split intensionality (b) World variables

Figure 2.2: Modifications of STI: wide-scope entails transparent evaluation while narrow-scope is compatible with transparent or opaque evaluation

is a region under ω where elements receive transparent evaluation; namely, the region between ω and OP$_\omega$. As illustrated in Figure 2.2-a, syntactic position still fully determines a DP's intensional status, but STI's biconditional dependence becomes a conditional one: wide-scope entails transparency, but not vice versa.

The same conditional dependence is the result of Percus's (2000) more radical departure from STI. In his system, world-dependent lexical items are functions whose input is a world-argument, saturated by silent object-language *world-variables* at LF. The restrictor of every DP is assumed to contain such a variable as its argument, which can be bound either by an intensional operator above it or by a binder posited at the matrix level, with the identity of the binder determining the intensional status of the DP. When a world-variable is bound by the matrix binder, the restrictor is transparent, and it is opaque when its variable is bound by the intensional operator above it. As shown in Figure 2.2-b, here, too, residing under ω does not determine a DP's intensional status, but being outside it does: ω cannot bind the world-pronoun of a DP above it, which must thus be evaluated transparently.

Both approaches, then, depart just enough from STI to generate the *third* reading, but not enough to allow opaque evaluation of wide-scope DPs, as is supposedly required for R4, presumably resulting in the undergeneration of any sentence that is judged true in an *exclusively*-R4-verifying scenario. Can we avoid this conclusion without rejecting the structural accounts of scope and binding? Perhaps, but our options seem limited. If Szabó's example truly involves a DP whose determiner takes wide-scope, that DP's restrictor cannot be interpreted opaquely unless, well, it does not serve to restrict the determiner it accompanies. Instead, this *pseudo*-restrictor must make its semantic con-

tribution within the intensional environment that their determiner outscopes. We explore this next.

Before we continue, a technical note: For convenience, we will be using a system that posits object-language world variables. However, to avoid clutter we sometimes omit these variables and their binders from LF representations when all intensional variables are assumed to be bound locally, by the binder immediately dominating them.

2.3 Szabó's solution and its problems

2.3.1 *Bare quantification*

In introducing their *Generalized Quantifier Theory* (henceforth, GQT), Barwise & Cooper (1981) famously observe that quantificational determiners in natural language are assumed to differ from predicate logic quantifiers ($\forall$, $\exists$) syntactically *and* semantically. Syntactically, they take a predicative complement, while predicate logic quantifiers combine with a formula. Semantically, their complement *restricts* their quantificational domain, whereas nothing restricts the domains of predicate logic quantifiers. Szabó argues that GQT got things wrong. While determiners always appear with predicates, they can quantify unrestrictedly like predicate logic quantifiers, with their complement predicate interpreted within their scope.

Szabó (2010) mediates the apparent mismatch between syntax, which requires determiners to take NP complements, and semantics, which allows them to quantify unrestrictedly, with a scope displacement operation, termed *quantifier splitting* (henceforth, QS) in (8), which allows the NP in the complement position of a raised DP to reconstruct into its pre-raising position without the DP's determiner. The determiner left in the derived position is restricted by an identity function, $\lambda x.x = x$ in (8), vacuously satisfying the syntactic requirement that determiners combine with predicates. The reconstructed NP is converted into an indexed definite description whose index is bound by the vacuously restricted determiner (cf. Fox 2003).[6]

(8) *Quantifier splitting (**QS**):*
$[_{\text{TP}}\,[_{\text{DP}}[D][_{\text{NP}}\,N]][[\lambda_\iota][_{\text{TP}}\ldots[_{\text{DP}}\,t_\iota]\ldots]]]$
$\rightarrow [_{\text{TP}}\,[_{\text{DP}}\,[D][\lambda x.x = x]]\,[[\lambda_\iota]\,[_{\text{TP}}\ldots[_{\text{DP}}\,\text{THE}\,[_{\text{NP}}\,N][\lambda y.y = x_\iota]]\,\ldots]]]$

By converting the reconstructed NP into an indexed definite description,

[6]Note that QS in (8) is a notational variant of Szabó's actual proposal, due to Benbaji (2021), which uses terms more familiar to linguists.

QS triggers a presupposition in the scope of the vacuously restricted quantifier in the derived position. We will make the simplifying assumption that this presupposition is locally accommodated.[7] Effectively, then, when a DP consisting of a quantificational determiner and its NP restrictor undergoes QS, elements quantified over by the determiner are asserted to satisfy the function denoted by NP inside the determiner's scope. Thus, if the determiner's scope itself contains an operator that shifts the evaluation domain of its input, this operator will also shift the evaluation domain of the reconstructed NP, resulting in exactly the configuration required to derive R4.

Now, Szabó (2011) is not explicit about how QS is applied to derive the R4 of (5b), so the following contains some guesswork on our part. However, his assumption that the truth of (5b) in the exclusively-R4-M-verifying scenario (5a) can be taken as evidence for a bare quantification strategy along the lines of (8), seems to imply that he views (5b) as containing a DP "*two judges*" that raises across the modal "*could*". In that case, indeed, QS derives for (5b) the structure schematized in (9a), with the modal scoping between the determiner and a "restrictor" NP converted into an indexed definite. With the presupposition of that indexed definite locally accommodated, (9a) has (9b) as its truth-conditions. Since the modal resides between the original and derived positions of the raised DP, it shifts the evaluation domain of the reconstructed restrictor without scoping above the determiner, resulting in wide-scope for the determiner and opaque evaluation of the restrictor.

(9) a.$[_{\text{TP}}$ there are $[[\text{DIST}][[_{\text{DP}}$ [two]$[\lambda x.x = x]][[\lambda_\iota][_{\text{TP}}$ could
[we face $[_{\text{DP}}$ [the]$[[\lambda y.y = x_\iota]$[judges]]] in court]]]]]]

b.$\lambda w.\ \exists_2 x\colon \forall y \in \text{ATOMS}(x)\colon \exists w' \in \text{ACC}(w)\colon$ we face the judge y in w'

2.3.2 Against *bare quantification*

There are two related reasons to reject QS as a possible scope displacement operation: First, it overgenerates, predicting R4s where they do not seem to be attested. In particular, Benbaji (2021) shows that R4-M seems restricted to particular relativization environments; namely, those in which the relevant determiner seems to take wide scope relative to a modal embedded in a relative clause (henceforth, RC) in its scope, headed by the NP that seems to be opaque relative to the RC-internal modal. This is illustrated by the modification of Szabó's modal case (5) in (10), as well as by the new example in (11).

[7] Benbaji (2021) shows that the arguments against QS cited below go through regardless of this.

(10) a. **Context:** Alex is awaiting the judicial elections, as in (5a) only this time there are five non-judges as candidates. Alex believes only two stand a chance, and instructs her lawyer to prepare for either winning.

b. There are two judges who could end up presiding over our case. **true** in (10a)

c. Two judges could end up presiding over our case. **false** in (10a)

(11) a. **Context:** A building collapsed, with only the security guard on the night shift present inside. There are only two people who are ever on duty at night, but we are not sure which of them picked up the shift on the night of the collapse. Thus, for all we know, each of them *might* be trapped under the rubble, but we are certain that it is not the case that both are.

b. There are two victims who might be trapped under the rubble. **true** in (11a)

c. Two victims might be trapped under the rubble. **false** in (11a)

The contexts in (10a) and (11a) above make *only* the R4 of their respective (b-c)-examples true. However, while the (b)-examples, which contain RCs, seem felicitous given the exclusively-R4-verifying context, the (c)-example, without relativization, seem false.[8] But QS generates a R4 for the (c)-examples, where it is widely assumed that the DP has raised over the modal, and can thus undergo the splitting operation, whose sole precondition is that such raising precede it.

Note that Benbaji (2021) uses the minimal pair in (10b-10c) rather than the one in (12a-12b) which contains Szabó's original sentence as one of its members, despite the fact that it gives rise to the same contrast as the one observed in (10), once the RC is eliminated from the sentence.

(12) a. There are two judges we could face in this court. **true** in (10a)

b. We could face two judges in this court. **false** in (10a)

This is because, one can plausibly argue that the contrast in (12a-12b) does not illustrate the central role of relativization in achieving R4-M. On the as-

[8] In a footnote, Szabó (2011) notes that according to his own judgment, a sentence like (10c) does have R4, though it is harder to tease apart than in cases with relativization, and an anonymous reviewer agrees. Several informants we consulted with judged the sentence as clearly out in an exclusively-R4-verifying scenario, and similar judgments about the general unavailability of R4 with modals are reported in Francez 2018. It is notable, in any case, that everyone in this debate seems to agree that it is at the very least much more difficult to achieve R4 without relativization than with it.

sumption that the existential DP and modal in these examples are scopally-commutative, discussed in Section 2.2.1, R4 of (12b) is only distinct from its *de dicto* reading when the plural DP scopes and *distributes* over the modal. But there is a known ban on or general dispreference for inverse-scope object-distributive readings, observed independently by, a.o., van der Does (1992); Steedman (2011); Križ & Maldonado (2018), which itself might explain the contrast in (12a-12b); i.e., perhaps QS is blocked independently in (12b) because the DP "*two judges*" cannot QR and distribute over the rest of the sentence. However, even if QS does not overgenerate a non-existent R4 for (12b) due to independent constraints on distributivity, it certainly overgenerates one for (10c) and (11c), and is thus *too expressive*.[9]

[9]Santorio (2013) independently argued that QS overgenerates, focusing on unattested fourth readings of attitude reports with a proportional quantifier—elements like *most, one fourth, less than half*—in its prejacent. Traditionally, these quantifiers are assumed to compare the cardinality of two sets. Allowing one of those sets to correspond to the entire domain, QS derives a true reading of Santorio's (a-ii) in the context of (a-i), contrary to fact: If the determiner "*less than half*" is vacuously restricted, while its NP complement "*stockbrokers*" is interpreted as part of its scope, (a-ii) is wrongly predicted to have the R4 truth-conditions in (a-iv), paraphrased in (a-iii).

(a) i. **Context:** Sam believes all stockbrokers are rich, and that stockbrokers constitute a minority of existing things. Moreover, she thinks non-rich things vastly outnumber rich ones.

ii. Sam believes less than half of the stockbrokers are rich. **false** in (a-i)

iii. Sam believes that less than half of the domain's entities are rich stockbrokers. **true** in (a-i)

iv. $\lambda w.\ \frac{|\{x \mid x=x\}|}{2} \geq |\{x \mid \forall w' \in \text{DOX}_{Sam,w} : x \text{ is a rich stockbroker in } w'\}|$

We are not sure whether this argument is conclusive, as there is some controversy regarding the scopal properties of proportional determiners, particularly those involving modified numerals like the one in (a-ii). The controversy pertains to the question of whether these determiners can take wide scope at all (cf. van der Does 1992; Ruys 1992; Beghelli 1993; Schein 1993; Reinhart 1997; Winter 1997), and that argument has not been decided. Of course, if it turns out that wide quantificational scope for the DP in (a-ii) is independently blocked—i.e., that "less than half of the stockbrokers" cannot QR in the first place— QS would not be expected to apply. There is some evidence that, indeed, the run-of-the-mill wide scope transparent reading of (a-ii) is independently hard to achieve. Some speakers reject (a-ii) in a scenario in which Sam thinks of certain actual stockbrokers that they are rich, but is either ignorant of the fact that they constitute less than half of the stockbrokers or thinks they constitute a majority of them. This suggests that wide scope might be blocked regardless of whether its restrictor is evaluated opaquely or transparently. We thank Kai von Fintel for this observation.

The argument in (10), we think, does not suffer from a similar objection, as nothing prevents a wide scope, *de re*, reading of (10c), but that sentence is nevertheless judged to be false in an exclusively-R4-M-verifying scenario.

This, in turn, leads us to the second problem with QS, which is—surprisingly—that QS is *not expressive enough* to generate a R4 for Szabó's original cases in (12a) or (10b). Once we notice the RC in the syntactic structure of these examples, we observe that QS actually *undergenerates* their observed R4 because, on a common assumption regarding the structure of RCs, discussed at more detail below, in (12a) the determiner above the RC and its adjacent NP—namely, "*two*" and "*judges*"—do not on their own constitute a DP that can undergo QS relative to the RC-internal modal "*could*". But the splitting operation in QS must be preceded by movement of a quantificational DP, so if "*two judges*" does not meet this description, the R4 of (12a) is undergenerated by QS.

These two arguments against Szabó's proposal are not of equal status. One is an argument against the very idea that natural language makes available a bare quantification strategy; if it did, we would observe fourth readings where we, in fact, do not. The other is an argument against a particular technical implementation of such a strategy; QS in particular, predicts that we should not observe particular fourth readings which, in fact, we do. Given that Szabó, is not committed to the technicalities of QS, which are provided "for the sole purpose of illustration" Szabó (2011: 270), we should wonder whether the latter challenge—from undergeneration—can be defused.

It seems to us that defusing it without undermining the case for bare quantification is a difficult task. For instance, one may attempt to widen the applicability of QS somehow so that it applies even for the cases it currently undergenerates. But doing so can only worsen the overgeneration problem QS faces. Alternatively, one may concede that the undergenerated cases of apparent R4-M are not derived via the same mechanism that derives bare quantification elsewhere. In that case, if we do not want to worsen the overgeneration problem, we must also concede that these cases are not instances of bare quantification at all. But then the claim that R4 is a general phenomenon, cutting across intensional environments, is undermined, as we lack an example of the reading relative to modal operators.

We argue next that the latter approach is correct; i.e., that since R4-M is contingent on relativization, its instances do not involve bare quantification. We will show that RCs already provide us with structures that can derive the attested cases of R4-M, and in all these cases, quantification *is* restricted, just not by the NP linearly succeeding the determiner; rather it is a reconstructed RC that restricts the determiner's domain of quantification.

2.4 Reducing R4-M to reconstruction effects

We have seen in (10-11) that R4 relative to modals, R4-M, is available only when the relevant modal is embedded inside a RC dominated by the DP with the apparent R4.[10] Next, we briefly review some structural properties argued to characterize RCs, and illustrate how several such properties by themselves derive the scopal configuration for which Szabó posits QS. As an independent mechanism, then, QS is *superfluous* in the derivation of R4-M. We then provide evidence that R4-M is indeed restricted to DPs with RCs that exhibit exactly those structural properties that derive R4-M.

[10] Szabó (2010) mentions the example in (i) that, prima facie, challenges this generalization, and a reviewer similarly suggests (ii).

(i) For her semantics seminar, Anna could write three papers. Now she has to decide which one to write.

(ii) You could build hundreds of different toys using the blocks in this LEGO set.

The sentence in (i), on its most natural interpretation, seems to convey that three things could be papers that Anna writes. Similarly, (ii) is judged true when hundreds of things could be toys built from the LEGO set, even if there are not enough blocks in the set to build hundreds of toys.

We think that the apparent R4-M of (i-ii) is due to their use of the creation verbs "write" and "build". R4-M seems unavailable for the variations in (i′-ii′), which—without a creation verb—have only a *de dicto* or *de re* reading. (ii′), for instance, seems committed to the claim that it is possible to sell hundreds of toys all formed from the discussed LEGO set. Note further, that in both cases, we can achieve R4-M by introducing a modal RC, as shown in (i″-ii″).

(i′) For her semantics seminar, Anna could read three papers.

(ii′) You could sell hundreds of different toys using the blocks in this LEGO set.

(i″) There are three papers Anna could read (depending on which of his ideas Ben plans to write about for his term paper).

(ii″) There are hundreds of different toys that you could sell using the blocks in this LEGO set.

This suggests that the apparent R4-M in (i-ii) should not be derived via a general mechanism dedicated to the derivation of R4-M, but rather from the semantics of creation verbs. Notably, on a prominent approach, such verbs are themselves treated as intensional operators, so that a sentence like 'John built a toy' asserts that a would-be toy has been instantiated in actuality by John (cf. Bennett 1977). But then, (i-ii) might just be true on their *de re* readings in context that would seem to verify their alleged R4-M: In (i) there are three *actual* would-be papers that Alex could instantiate by writing, and in (ii) there are hundreds of actual would-be toys that could each be instantiated using the LEGO set. We can thus tentatively conclude that (i-ii) should not be taken to bear on our theory of R4-M.

2.4.1 Raising relative clauses

Brame (1968) first suggested that RCs have LFs in which the head NP originate RC-internally (see also Schachter 1973; Vergnaud 1974, and Kayne 1994). These raising LFs are posited to derive the observation that some grammatical RCs comply with independent syntactic conditions *only* if their heads originate in RC-internal positions. For instance, the head-NP in (13a) contains an anaphor which satisfies Binding Condition A only when bound by a binder that resides inside the RC. The anaphor must have therefore originated below its binder, and binding conditions must have been "checked" before it raised to its RC-external position. Relatedly, in (13b) the head forms an idiom with an embedded VP. Assuming that the verbal argument in VP idioms must reside in the verb's complement position at LF, this indicates that the RC-head had raised from VP's complement position. Crucially, then, the examples in (13) are grammatical because they can be parsed as if the RC-head is interpreted in its pre-raising position.

(13) RCs *only* compatible with a raising structure:

a. Condition A: $[_{\text{DP}}$ D $[_{\text{NP}} \ldots \text{anaphor}_i \ldots]_j$ $[_{\text{RC}} \ldots \text{R-expression}_i \ldots \text{t}_j]]$

i. The rumor about himself$_i$ that John$_i$ denied is unflattering.

ii. The interest in each other$_i$ that John and Sue$_i$ showed was fleeting.

b. Idioms: $[_{\text{DP}}$ D $[_{\text{NP}}$ idiom part$]_j$ $[_{\text{RC}} \ldots$ idiomatic environment $\text{t}_j \ldots]]$

i. John kept tabs on his boss.

ii. Mary could not ignore the tabs that John kept on his boss.

The two properties in (14) often attributed to such *raising* relative clauses (henceforth, RRCs) will be taken for granted in our discussion. Both assumptions have been defended on independent grounds: (14a) is required for type reasons (Heim & Kratzer 1998: 96-8), and to account for the behavior of RRCs under presuppositional determiners (Partee 1975: 230-1).[11] More controversially, (14b) is motivated by various syntactic arguments, including the data in (13): Reconstruction of the head into the RCs in (13) is required to explain their apparent compatibility with syntactic well-formedness conditions.[12]

[11] See also Heim 1987: 36-37 and Bhatt 2002: 70-71 for further arguments. In particular, Heim shows that RCs under a strong determiner in the sense of Milsark 1974 can have their head originate in the post-copular position of a *there*-construction embedded within the RC. Since strong determiners are barred from this position, the determiner itself could not have originated RC-internally.

[12] See Bhatt 2002 for a critical assessment of these arguments, and for additional ones. The controversy regarding syntactic reconstruction is beyond our focus, though we briefly remark in Footnote 24 on how R4 might bear on it.

Note, also, that the property in (14a) is what prevents QS from deriving R4-M for Szabó's example in (12a): The determiner and adjacent NP in that example originate in different structural positions, and thus cannot constitute the input to QS to begin with.[13]

(14) a. A determiner under which a RRC is embedded originates RC-externally, unlike the head NP. Thus, RRC-embedding DPs have the following structure: $[_{DP}\,[D][[NP][_{RC}\ldots\,[\overline{NP}]\ldots]]]$.

b. The head NP, which originates RC-internally and raises to its surface position, can *reconstruct* into its pre-raising position at LF.

Building on Heim 2018, Bassi & Rasin (2018) propose the three-step derivation in (15) for RRCs with a reconstructed head NP.

(15) a. *Merge copy of NP ("raising")*:
$[_{RC}\ldots\,[_{DP}[D][_{NP}\,N]]] \rightarrow [[_{NP}\,N][[\lambda_i][_{RC}\ldots\,[_{DP}[D][_{NP}\,N]]]]]$

b. *Trace conversion:*

i. $[[_{NP}\,N][[\lambda_i][_{RC}\ldots\,[_{DP}[D][_{NP}\,N]]]]$
$\rightarrow [[_{NP}\,N][[\lambda_i][_{RC}\ldots\,[_{DP}[D][_{NP}\,[N][\lambda y.y = x_i]]]]]]$

ii. $\rightarrow [[_{NP}\,N][[\lambda_i][_{RC}\ldots\,[_{DP}[\textsc{the}][_{NP}\,[N][\lambda y.y = x_i]]]]]]$

c. *Higher copy LF-deletion ("reconstruction"):*
$[[_{NP}\,N][[\lambda_i][_{RC}\ldots\,[_{DP}[\textsc{the}][_{NP}\,[N][\lambda y.y = x_i]]]]]]$
$\rightarrow [[\lambda_i][_{RC}\ldots\,[_{DP}[\textsc{the}][_{NP}\,[N][\lambda y.y = x_i]]]]]$

The proposal treats raising as the merging of copies, and reconstruction as the deletion of certain copies at LF. Step (15a) thus involves merging a copy of the NP restrictor of a RC-internal DP in a RC-external position, together with a λ-binder directly below it. Next, the RC-internal DP whose restrictor has moved undergoes *trace conversion* (15b); i.e., it is converted into an indexed definite description whose index is bound by the λ-binder introduced by raising (Fox, 2003). Finally, the higher *pronounced* copy of NP is ignored for interpretative purposes (15c). Note that, for brevity, we will sometime use the long-form $[_{DP}[\textsc{the}][_{NP}[N][\lambda y.y{=}x_i]]]$ to represent trace-converted DPs, and sometimes its shorter counterpart $[_{DP}[\textsc{the}_i][NP]]$.

[13] Moreover, while (14a) is a property unique to *raising* RCs, QS would also fail to apply in the desired way if the RC in (12a) were not analyzed as involving raising. On a particular version of the non-raising structure known as the "matching" construal, the determiner and NP do form a DP on their own, but that DP originates RC-externally as a whole, and QS would therefore never be able to place the NP in a RC-internal position below the modal, as would be required to achieve R4-M.

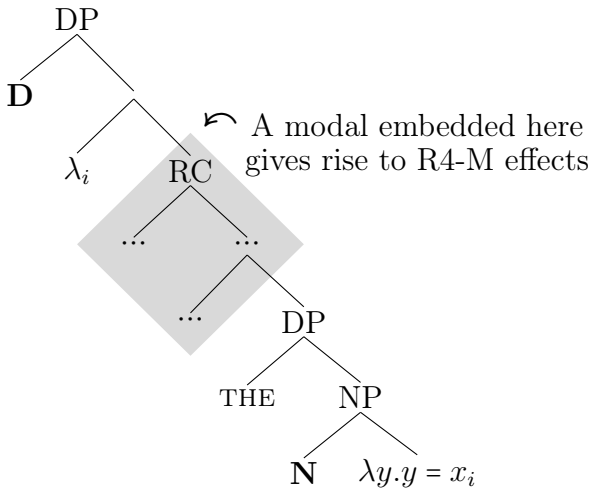


Figure 2.3: Structure of DPs embedding reconstructed RRCs

The properties in (14), together with the account in (15), derive the structure in Figure 2.3 for DPs embedding a reconstructed RRC. Crucially, the structural relationship between the RC-external determiner and the RC-internal NP, both boldfaced in the Figure, is exactly the relationship between the determiner and the nominal complement of a DP that has undergone Szabó's QS (8). When an intensional operator is situated between a DP's original and derived positions, QS generates R4. However, given our observation that R4-M in particular seems restricted to the kind of structures schematized in Figure 2.3, QS is unnecessary: Whenever a modal is part of the shaded region in the figure, R4-M comes for free, simply in virtue reconstruction.

Less abstractly, Szabó's example of R4-M (16) takes the form of a *there*-sentence (Milsark, 1974) with a numeral in its post-copular position. For ease of exposition, we treat a numeral n as an existential quantifier over n-sized pluralities that can distribute over its restrictor argument. This is implemented below by having a distributivity operator DIST (7) adjoin the restrictor of n. The *there*-sentence as a whole is then assumed to behave as if its post-copular DP has vacuous scope; i.e., asserts the existence of a n-sized plurality whose atoms *each* make the embedded NP true, as in (17).[14]

[14]Distributivity is necessary, though not *this* implementation (cf. Section 2.2.1). Both our treatment of numerals as determiners, and our assumptions regarding the meaning of Milsark's *there*-construction, are not essential, but are rather useful simplifications. We could have easily assumed, with, e.g., Keshet (2010a) that *there*-sentences involve existential closure over a predicative element, and a numeral n in the post-copular position is a predicative element that restricts this existential quantification which can distribute over

(16) There are two judges we could face in court.

(17) $[\![[\text{There are } [_{\text{DP}}\ [n\ [\text{DIST NP}]]]]]\!] = \lambda w.\ \exists_n x : \forall y \in \text{ATOMS}(x) : [\![\text{NP}]\!](y)(w)$

Following the recipe in (15), we construct the LF structure in (18a) for the underlined NP in (16). This LF derives the set of individuals for whom there is an accessible world in which they are the judge Alex faces in court (18b). The sentence thus asserts that there are two individuals, each of whom could be the judge Alex faces in court. This, of course, is the R4 paraphrase of (16).

(18) a. $[[\lambda_i][_{\text{RC}}\ [_{\text{TP}}\ [\text{could}]\ [_{v\text{P}}\ \text{we face}\ [_{\text{DP}}[\text{THE}_i][_{\text{NP}}\ \text{judge}]]\ \text{in court}]]]]$

b. $[\![(18a)]\!] = \lambda x \lambda w.\ \exists w' \in \text{ACC}(w)$:
we face in court in w' the unique y who is a judge in w' s.t. $x = y$

Note that on this analysis, both the choice of determiner in (16) and its *there*-sentence structure, are entirely orthogonal to the availability of R4-M. All that is required for R4-M is that the determiner, *whichever it is*, embed a RRC with a modal outscoping the original position of its head-NP. That this is indeed the case is illustrated by the examples in (19), which do not have the structure of an existential *there*-sentence and whose determiners are not numerals but are nevertheless able to quantify over potential, rather than actual judges.

(19) a. **Context:** Alex's lawyer obtains a list of potential judicial election candidates and presents it to Alex.

b. If the list exhausts all potential candidates, the lawyer can felicitously utter: "*Every judge we could face in court is on this list.*"

c. If Alex thinks the list does not in fact contain any potential candidates, she can felicitously utter: "*No judge we could face in court is on this list.*"

d. If the list contains some but not all potential candidates, the lawyer can felicitously utter: "*A judge we could face in court is on this list.*"

e. If the lawyer is certain that the eventual winner is listed in the document, they can felicitously utter: "*The judge we will face in court is on this list.*"

(adapted from Benbaji 2021: ex. 22)

material below it. We only crucially assume that a numeral may originate RC-externally even when an NP below it heads a RC.

2.5 Reconstruction as a precondition for R4-M

Let us take stock: In previous sections we make several claims. We argue that bare quantification should not be generally available, given its overgeneration problem, and is also not needed to derive R4-M, given that R4-M is restricted to relativization environments and can thus, in principle, be derived via reconstruction. We now want to justify a stronger thesis; namely, that R4-M *must* be derived as a side-effect of reconstruction. If this is the case, we predict the reading to be blocked whenever reconstruction is ruled out independently. The literature provides several diagnostics for the absence of reconstruction, which—we show next—correlate in the expected way with the unavailability of R4-M.[15] It is important to note that this next part of the chapter no longer takes Szabó's argument for bare quantification as its foil; rather, it can be viewed as a "sanity check" for the idea that reconstruction in particular is in charge of deriving R4-M when this reading is attested.

2.5.1 Resumption

Sichel (2014) shows that Hebrew direct-object resumptive pronouns (hereafter, RPs) in RC-internal positions block reconstruction. This is illustrated by the illicitness of such RPs in environments that *require* reconstruction. In (20), for instance, reconstruction is required for an anaphor to satisfy Condition A.

(20) ha-ʃmu'a al acmo$_i$ ʃe-Dani$_i$ hikxiʃ (*ota) hufca al yeday Rani
The-rumor about himself that-Dani denied (*it.RP) was.spread on hands.of Rani
'The rumor about himself that Danny denied was spread by Rani.'
(Sichel 2014: ex. 4)

[15] The ability to capture the correlation between R4-M and reconstruction constitutes, we think, a point in favor of our proposal compared to an alternative truth-maker semantic account, due to Moltmann (2019, 2020b), who derives Szabó's data by positing a domain of *variable-objects*, each associated with their own concept-like function from *minimal-situations* to *manifestations-in-a-situation*. While these variable-objects may have different manifestations in different situations, they are nevertheless *objects* in the ontology, rather than functions. If we understand the proposal correctly, Szabó's example of R4-M (16) is satisfied by there being in our domain two variable-objects that each have a manifestation as a judge Alex faces in some possible court situation. It seems to us that such a system can explain the correlation of R4-M and reconstruction only by positing distinct constraints on variable-object-denoting expressions that tie them to the structure of the sentences in which they appear. But to the extent that such constraints can be formulated, one wonders whether the resulting system would not converge with the account provided here.

In (21), it is required to preserve the integrity of the idiom "*sew a case*", meaning *to falsely implicate in a crime*. Both environments disallow RPs.

(21) ha-tik ʃe-tafru (#oto) la-sar haya kaʃur
the-case that-they.sewed (#it.RP) for.the-minister was related
le-nadlan
to-real.estate
'The case that they pinned on the minister was related to real estate.'
(Sichel 2014: ex. 5a)

Benbaji (2021) already illustrates that direct-object RPs block R4-M, as predicted by the reductive approach. The Hebrew translation of Szabó's example with a RP in (22) is infelicitous in Szabó's exclusively-R4-M-verifying scenario (5a), as it seems to commit one to the existence of *actual* judges.

(22) yeʃ ʃney ʃoftim ʃe-ʔanu ʔalulim lifgoʃ (#ʔotam) bevet
there.are two judges that-we could meet them.RP in.house
hamiʃpat
the.law
'There are two judges we could face in court.' (Benbaji 2021: ex. 24)

However, the reductive approach makes another, previously unexplored prediction regarding resumption: While optional direct-object RPs block reconstruction, obligatory *indirect*-object RPs allow it. Thus, Condition A is satisfied in (23), and idiomatic interpretation of the phrase "*climb on/descend from a tall tree*" (which, roughly, mean *to insist/relinquish an entrenched position*) is available in (24), despite their RPs.

(23) ha-ʃmu'a al acmo$_i$ ʃe-Dani$_i$ xaʃaʃ *(mimena) hufca
The-rumor about himself that-Dani feared from.it.RP was.spread
al yeday Rani
on hands.of Rani
'The rumor about himself that Danny feared was spread by Rani.'
(Sichel 2014: ex. 9)

(24) a.ha-ec ha-gavoha ʃe-hu tipes *(alav)
The-tree the-tall that-he climbed on.it.RP
'The position that he was stubbornly entrenched in'

b.ha-ec ha-gavoha ʃe-hu yarad *(mimeno)
The-tree the-tall that-he descended from.it.RP
'The entrenched position that he relinquished'
(adapted from Sichel 2014: ex. 10a-b)

Strikingly, RP-position correlates with the un/availability of R4-M in the way one would expect if R4-M is a byproduct of reconstruction. The variant of (22) in (25), in which the head NP "*judges*" raises from an indirect- rather than direct-object position of the RC-internal verb, is felicitous in the exclusively-R4-M-verifying scenario, despite the RP.

(25) yeʃ ʃney ʃoftim ʃe-ha-goral ʃeli bevet hamiʃpat ʔalul
there.are two judges that-the-fate mine in.house the.law could
lihyot taluy *(bahem)
be dependent on.them.RP
'There are two judges my fate in court could depend on.'

This is expected on the reductive approach as, given that reconstruction is *not* blocked by indirect-object RPs, nothing should block the effect of R4-M.

2.5.2 Extraposition

Reconstruction is also argued to be blocked for RC adjuncts extraposed over a right-adjoined adverb. Hulsey & Sauerland (2006) illustrate this by attempting to extrapose RCs that comply with independent syntactic constraints *only* if their head reconstructs into a RC-internal position. The resulting ungrammaticality or infelicity in (26) suggests that reconstruction, required for the satisfaction of Condition A in (26a) and to achieve an idiomatic interpretation of (26b), is blocked by extraposition.

(26) a. * I saw the portrait of himself$_i$ yesterday that John$_i$ likes.[16]
b. # Mary could not ignore the tabs yesterday that John was keeping on her.

As predicted, R4-M is indeed blocked when the modal-embedding RC is extraposed. In the exclusively R4-M-verifying scenario (27a), the example without extraposition seems true (27b), whereas its counterpart with it in (27c) is judged false.

[16]This is Hulsey & Sauerland's example. As Isabelle Charnavel and Dominique Sportiche (p.c.) note, some speakers may judge the sentence to be better than predicted if extraposition indeed blocks reconstruction, given that animate anaphors are sometimes exempt from binding condition A (cf. Charnavel & Sportiche 2016; Charnavel 2019). To avoid this confound, we may consider the contrast in (i), where an inanimate anaphor is used.

(i) a. I noticed the citation of itself$_i$ that the paper$_i$ mentions in a footnote.
b. * I noticed the citation of itself$_i$ yesterday that the paper$_i$ mentions in a footnote.

(27) a. **Context:** Alex is again awaiting the results of the judicial elections, whose candidates are again all non-judges, to see who will preside over her court case. She is a well-known politician and is encouraged by her lawyer to use her political platform to praise all judicial candidates, so that whoever wins will judge her favorably. She says:

b. I praised every judge that we could face in court. **true** in (27a)

c. I praised every judge yesterday that we could face in court. **false** in (27a)

(adapted from Benbaji 2021: ex. 27)

This falls out from the reductive account, of course, if extraposition blocks reconstruction, which is a precondition for R4-M.

2.5.3 Adjectival modification of the head noun

Next, we explore a consequence of the reconstruction account of R4-M, when combined with a particular hypothesis regarding the way reconstruction in raising RCs interacts with adjectival modification, and an independently motivated constraint on the evaluation world of modifiers. Unlike the previous predictions we tested, this one requires a more involved introduction.

The hypothesis that we will explore here states that raising RCs with adjectival modification, i.e., those with the surface form in (28), may be mapped only to a restricted set of LF structures in which the adjective and the noun are interpreted in the same position. This means that when independent considerations force an adjectival modifier to be interpreted RC-externally, the head noun will not be able to reconstruct, ruling out the possibility of R4-M.

(28) Det Adjective Noun RC

More precisely, the hypothesis states that the range of possible structures one could assign to a DP with the surface form in (28) is in (29).

(29) a. $[_{DP}\,[D][[_{NP}\,AP\ N][_{RC}\ldots[_{NP}\,\sout{AP\ N}]\ldots]]]$

b. $[_{DP}\,[D][[_{NP}\,\sout{AP\ N}][_{RC}\ldots[_{NP}\,AP\ N]\ldots]]]$

c. * $[_{DP}\,[D][[_{NP}\,AP\ \sout{N}][_{RC}\ldots[_{NP}\,\sout{AP}\ N]\ldots]]]$

d. $[_{DP}\,[D][[AP]\,[[_{NP}\,N][_{RC}\ldots[_{NP}\,\sout{N}]\ldots]]]]$

e. * $[_{DP}\,[D][[AP]\,[[_{NP}\,\sout{N}][_{RC}\ldots[_{NP}\,N]\ldots]]]]$

The adjective can originate with the noun it modifies inside the RC, and raise with it to be interpreted RC-externally (29a), or reconstruct with it to be

interpreted RC-internally (29b). The adjective may also merge RC-externally, if the noun itself is interpreted in its derived, RC-external position (29d). The hypothesis rules out structures in which the adjective raises with the noun, but reconstructs without it (29c), and those in which the adjective originates RC-externally, but the noun reconstructs to be interpreted RC-internally (29e).

This hypothesis—ruling out structures with partial reconstruction (29c) as well as structures that mimic the interpretative effects of partial reconstruction (29e)—seems to us quite intuitive. But our goal here is not to motivate it.[17] Rather, what we will show is that, when paired with the reconstruction view of R4-M and an independently motivated generalization about the evaluation world of modifiers (to be introduced next), this hypothesis makes exactly the right predictions with respect to the distribution of R4-M; i.e., that it is in fact the case that interpreting an adjective RC-externally blocks R4-M with respect to the head noun.

[17]For completeness, this footnote offers a potential account; namely, a set of selection and labeling principles that conspire to give us the picture in (29). Note that this is done for illustrative purposes only, as the literature on the interaction between labeling and selection in the context of RC structure is quite vast, and we cannot do justice to it within the confines of the present discussion (see, for instance Cecchetto & Donati 2015; Salzmann 2017 for some recent work and comprehensive bibliographies).

Consider the following principles:

(i) Determiners select either for NPs, or for constituents labeled with whatever label is associated with RCs, let us assume that this is CP.

(ii) the label on the constituent containing the RC and its head NP depends on whether the head reconstructs: with reconstruction, that constituent inherits the label of the RC—namely, CP—but without reconstruction, it inherits the label of the head NP.

(iii) NPs can have APs as their adjuncts, but crucially CPs cannot.

In (29a), the adjective is an adjunct of a NP, satisfying (iii), the modified NP does not reconstruct and so the constituent that dominates it inherits its NP label according to (ii), and so the determiner is merged with a NP, satisfying (i). In (29b), since the modified NP reconstructs, the constituent that combines with the determiner inherits the label of the RC, namely CP, and this is also permitted by (i), resulting in an acceptable structure. In (29d) the adjective does not originate as a modifier of the head NP in its RC-internal position. However, the structure is acceptable because the raising NP does not reconstruct, so the constituent dominating it in its derived position is itself a NP that can have an AP adjunct. The ungrammatical structures are ruled out as follows: In (29e) the adjective originates RC-externally, but the head noun reconstructs to its original, RC-internal merging position. Thus, (ii) dictates that the constituent that AP is merged as an adjunct of inherits the RC's CP label, violating (iii). The structure in (29c) is not ruled out by the principles in (i-iii), but rather by the reconstruction algorithm in (15); reconstruction is the deletion of the higher copy of the moved phrase, and the moved phrase in (29c) contains the adjective *and* the noun, which can thus only be deleted together.

Intersective modification

Keshet (2010a) shows that two predicates interpreted intersectively must have the same intensional status, motivating the generalization in (30). This is illustrated with the infelicity of the sentences in (31), which involve NPs consisting of a noun and a modifier that contradict each other, and should have been felicitous if the contradicting noun and modifier could have different evaluation worlds.[18]

(30) **The intersective predicate generalization:** Two predicates interpreted intersectively may not be evaluated at different worlds from one another.

(31) a. # Mary thinks the married bachelor is messy.

b. # Mary thinks the professor in college is too young to teach.

(Keshet 2010a: ex. 19)

With this in mind, consider a sentence ϕ containing a DP with the surface form in (28), in which the adjective involved is intersective,[19] and the RC involved contains a modal. And assume further that ϕ is uttered in a scenario that verifies it only if the noun is evaluated opaquely relative to the RC-internal modal while the intersective modifier is evaluated transparently. In other words, the background context verifies only a *de re* reading of the modifier part of the RC-head, but only the R4-M of the noun that it modifies.

But the structures for the RC-embedding DP that could, in principle, derive such a reading for ϕ, are all ruled out by the independent assumptions introduced above. One such structure has the head noun raise with its adjectival modifier from a RC-internal position, but reconstruct without it (32a). Another has the modifier originate RC-externally, but has the head noun reconstruct (32b). These are ruled out by the hypothesis that structures with

[18] See Keshet 2010a for further data in support of the generalization, and von Fintel, Fox, & Iatridou 2014 for a critical discussion.

[19] Note that on a particular view of adjectival modification, the class of adjectives that count as intersective for our purposes is larger than might seem at first. *Gray* is intersective: Being a *gray elephant*, entails being gray and being an elephant. *Small* seems non-intersective: being a *small elephant*, does not entail being small and being an elephant; if it did, being a small elephant would be wrongly predicted to entail being small. Rather, *small* belongs to a class of adjectives often termed *subsective* (Partee, 1995; Kennedy, 2012). However, as Heim & Kratzer (1998) and Larson (1998) suggest, if subsective adjectives are assumed to come with a contextually supplied comparison class, then they, too, become intersective: If *small* is taken to be true of an individual x iff x is small given a contextually salient comparison class C, then being a *small elephant* does entail being small (given the comparison class) and being an elephant. On this view Keshet's generalization should apply widely, even to so-called subsective adjectives.

partial reconstruction and those mimicking them are ungrammatical. A final structure that would derive such a reading involves raising and reconstruction of the entire modified NP, but evaluation of the noun and the adjective in different evaluation worlds (32c). This structure, in turn, is ruled out by Keshet's in (30). We thus predict ϕ to be judged as false in a scenario that renders true only a *de re* reading of the modifier part of the RC-head, but only the R4-M of the noun that it modifies, due to a combination of Keshet's generalization and the hypothesis spelled out above.[20]

(32) a. * $[\lambda w \ldots [_{\text{DP}} [\text{D}][[_{\text{NP}} \text{AP}_{w} \sout{\text{N}}][_{\text{RC}} \ldots [\text{Modal } \lambda w' \ldots [_{\text{NP}} \sout{\text{AP}} \text{N}_{w'}] \ldots]]]]]$

b. * $[\lambda w \ldots [_{\text{DP}} [\text{D}][[\text{AP}_{w}] [[_{\text{NP}} \sout{\text{N}}][_{\text{RC}} \ldots [\text{Modal } \lambda w' \ldots [_{\text{NP}} \text{N}_{w'}] \ldots]]]]]]$

c. * $[\lambda w \ldots [_{\text{DP}} [\text{D}][[_{\text{NP}} \sout{\text{AP N}}][_{\text{RC}} \ldots [\text{Modal } \lambda w' \ldots [_{\text{NP}} \text{AP}_{w} \text{N}_{w'}] \ldots]]]]]$

The prediction seems to be borne out. Consider, for instance, the sentence in (33b) relative to the context in (33a). That sentence is quite odd in this scenario, exactly because it can be true in that scenario only if the modifier in the RC head is transparent, while the noun itself is reconstructed to achieve R4-M: The universal quantifier must quantify over judicial candidates, not over actual judges, who are poor in actuality, but not in those worlds in which they win the judicial election. Such a reading for (33b), paraphrasable as in (33c), requires an illicit structure, hence the oddity of (33b) in the above scenario.

(33) a. **Context:** Alex is accused of stealing from the rich to give to the poor. An election will determine which of ten judicial candidates (none of whom is or has ever been a judge) will preside over her court case, and Alex wants her lawyer to frequent social gatherings that those candidates attend, in an attempt to gain their sympathy. However, she thinks that candidates who are themselves among the

[20] As a reviewer points out, Keshet (2010a) does not explicitly discuss violations of (30) in cases that require reconstruction. However, we can show that the generalization regulates such cases as well. For instance, the example in (i) is as infelicitous as those in (31), even when the indefinite DP "*a married bachelor*" reconstructs under the raising predicate "*appear/seem*" for scope reasons. For instance, suppose we are allowed to peep into the apartments of ten men who we know are married, without knowing which apartment belongs to which man, and observe that one of the apartments is as messy as a bachelor pad. We cannot express with (i) the proposition that it appears that an actually married man is a messy bachelor, as predicted if (30) applies.

(i) #A married bachelor appears/seems messy.

rich will never be sympathetic to her cause, and wants her lawyer to focus on the poor candidates, even though she knows that whoever ends up winning—rich or poor—will not be poor when deciding her case, given the high salary judges are paid. She also thinks that only five of the ten candidates might actually win the election, three of which are poor. She finds out that these three candidates will be attending a party, and tells her lawyer: You must attend that party...

b. Every poor judge that we could face in court will be there. **false** in (33a)

c. Every poor candidate that we could face as the judge in court will be there. **true** in (33a)

2.5.4 Superlative modifiers

Bhatt (2002) observed that a RC whose head contains a superlative modifier is ambiguous between a 'low' reading in which the modifier is interpreted under RC-internal material, and a 'high' one in which it is interpreted RC-externally: If the books Sam attributes to Tolstoy are *Anna Karenina* (AK) and *War and Peace* (WP), (34) has both a true and a false reading. Since WP is, in fact, longer than AK, the *high* reading in (34a) is false. But if Sam (wrongly) claimed AK is longer, the *low* reading in (34b) is true.

(34) The longest book that Sam said Tolstoy wrote is Anna Karenina.

a. '*high-reading*': Of the books Sam said Tolstoy wrote, Anna Karenina is the longest.

b. '*low-reading*': Sam said the following: 'Anna Karenina is the longest book Tolstoy wrote'.

Bhatt attributes this ambiguity to reconstruction as well, arguing that the low-reading is obtained when the superlative RC-head reconstructs into the scope of the RC-internal attitude, while the high-reading is obtained when it is interpreted RC-externally. However, the derivation of the low-reading by means of reconstruction is the subject of much controversy in the literature; there is some doubt whether reconstruction should be appealed to in the first place, and even within the pro-reconstruction camp, the structure of the low-reading LF is up for debate.[21] Even without committing to a particular position in these debates, our reductive approach to R4-M makes a non-trivial

[21] Heycock (2005) argues against the reconstruction account, suggesting that the low-reading is restricted in ways it cannot explain. Bhatt & Sharvit (2005); Hulsey & Sauerland

prediction with respect to Bhatt's ambiguity. This is because, while we may argue whether the low-reading *necessitates* reconstruction, there seems to be a consensus that the high-reading *disallows* it. If this is correct, then under the hypothesis we introduce above regarding partial reconstruction, we predict R4-M to be ruled out when a high-reading is *forced*.

The scenarios that render the high and low readings of (34) true differ in whether the embedded subject is taken to have made any claims about the *lengths* of Tolstoy's books. The high-reading can be true even if Sam has made no such claims whatsoever, whereas its low-reading is only true if Sam made a particular claim; namely, that Anna Karenina is the longest of Tolstoy's books. Thus, in the LF for the *high* reading, the superlative operator and the adjective itself must be interpreted outside the intensional environment created by the propositional attitude. Otherwise, the truth of the sentence would hinge on Sam having ascribed particular lengths to those books she attributes to Tolstoy. In other words, the high-reading is incompatible with reconstruction, at least of the superlative and its adjective. Now we can appeal again to the hypothesis that interpretation of the adjective in an RC-external position necessitates interpretation of the head noun in that same position, to formulate a prediction regarding the availability of R4-M. If the high reading prohibits reconstruction of the adjective, it should be incompatible with R4-M.

Bhatt provides us with a tool to disambiguate the two readings. In particular, he observes that a *negative polarity item* (NPI) intervening between the RC-internal attitude and the modified NP-head has the following effect: A NPI (like '*ever*') above the attitude *necessitates* a high-reading of the sentence (35a), and *blocks* it when it is below the attitude (35b).

(35) a. The longest book that Sam **ever** said Tolstoy wrote is AK.

b. The longest book that Sam said Tolstoy **ever** wrote is AK.

What exactly these facts are supposed to teach us is, again, subject to some debate; particularly regarding the unavailability of the high-reading in (35b).[22]

(2006) and Sharvit (2007) offer counterarguments, but differ in their proposed reconstruction analyses. Charnavel (2022, 2024, 2026) provides a new perspective on the debate, as well as a useful summary of the pros and cons of reconstruction.

[22] See in particular the exchange in Bhatt 2002; Bhatt & Sharvit 2005; Heycock 2005, 2019. Bhatt, as mentioned above, attributes the high/low ambiguity to reconstruction. He thus explains the NPI facts as follows: Assuming NPIs are licensed only in the scope of downward-entailing operators (Ladusaw, 1979), and that the superlative is the only downward-entailing operator in (35), the low-reading is blocked for (35a) because it requires reconstruction of the superlative to a position below "*ever*", leaving the NPI unlicensed. In (35b), "*ever*" is in the scope of the superlative even after reconstruction, licensing the low-reading.

However, we can once again shy away from the controversy in testing our prediction *vis-a-vis* R4-M and NPI placement: Given that a high '*ever*' forces a high-reading, which in turn, disallows reconstruction of the head noun, a high '*ever*' should block R4-M.

That this is indeed the case can be illustrated with the contrast in (36).

(36) a. The richest judge Sam **ever** said we could face in court is the Libertarian.

b. The richest judge Sam said we could **ever** face in court is the Libertarian.

The NPI in (36a) forces a high-reading of the superlative operator and its adjective in the RC-head, and also appears to render unavailable the R4-M of the sentence: (36a) seems unable to convey that the richest individual Sam ever said we could face as the judge in court is the libertarian. This is exemplified by the fact that the sentence seems false in a scenario like (37), which involves no actual judges and thus verifies only its R4-M. On the other hand, when the NPI is low as in (36b), the sentence seems true in such an exclusively-R4-M-verifying scenario.[23]

(37) **Context:** As in (33a), Alex is facing trial for stealing from the rich to give to the poor. She again wants her lawyer to connect in person with judicial candidates (none of whom is or has ever been a judge), but is now unsure which of the candidates stands a chance to win, and which of them are too rich to ever be sympathetic to her cause. To avoid wasting time on hopeless candidates, she asks Sam, a consultant, for guidance. Sam suggests that only a Libertarian and two Democrats are realistic candidates, noting that the libertarian is the richest among them. Alex takes Sam to be an authority on these matters, and accepts her word as the truth.

This illustrates that the high-reading is incompatible with R4-M, which again reinforces the correlation between R4-M and reconstruction, on the assumption that the high-reading itself is incompatible with reconstruction.[24]

[23] We have so far intentionally remained agnostic about the relation between the low-reading and reconstruction. Note for completeness, however, that since we take reconstruction to be a necessary ingredient of R4-M, and we observe that a sentence like (36b) is felicitous in exclusively R4-M-verifying scenarios, we can no longer remain fully agnostic with respect to this issue. If R4-M is parasitic on reconstruction, than we are committed to the low-reading being derivable in principle from an LF with a reconstructed RC-head. This is a different claim from the one made in Bhatt 2002, according to which the low-reading is an argument *for* the availability of reconstruction, and a less controversial one at that (cf. Heycock 2019).

[24] This concludes our illustration of the correlation between reconstruction and R4-M.

2.6 Potential overgeneration

Above, we advance the claim that R4-M is restricted to reconstructed RCs. Next, we address a potential overgeneration challenge due to an anonymous reviewer, suggesting that our account, while less permissive than Szabó's (2010; 2011) bare-quantification approach, is still *too permissive*; there are structures in which reconstruction is presumably permitted, which nevertheless do not give rise to R4-M. These involve RCs whose head-NPs are not bare nouns, but rather modified by certain PPs or by multiple RCs, which seem to resist the reconstruction required to achieve R4-M. The response we offer will not be definitive. Rather, we will attempt to sketch the facts, as judgments are quite delicate, and then contemplate about what might account for the complex empirical picture.

Consider the context in (38a), which verifies, exclusively the R4-M of the sentences below. Given that on the other side of the courtroom door there is but one judge waiting to face Alex, the paper handed to her cannot possibly list six such judges. Rather, it lists six individuals who each could be that one judge. The sentence in (38b) seems true in this context, as it seems to have a R4-M paraphrasable as in (38e). The sentences in (38c-38d), on the other hand, seem false, suggesting they lack that reading.

(38) a. **Context:** A judicial election has occurred; of ten candidates, neither of whom were previously judges, one prevailed and filled a vacancy on the court. Alex will face that winner in an upcoming trial, but was not informed on the outcome ahead of time. As she sits outside the courtroom with her lawyer she wants to refresh her memory regarding who she might face. Her lawyer does not remember the exhaustive

We would be remiss, however, if we do not comment on the debate about the nature of reconstruction, and how the current discussion interacts with it. The *syntactic* account of reconstruction that we adopt in (15) contrasts with its treatment as a *semantic* process, consisting of type-shifting and abstraction over higher-type variables (e.g., Sharvit 1999). In fact, Grosu & Krifka (2007) defend a semantic account of reconstruction in exactly the RCs discussed here, whose head is interpreted opaquely relative to a RC-internal intensional operator. Our approach to R4-M does not bear directly on this issue. That is, to the extent that semantic reconstruction accounts for the way RCs pattern *vis-a-vis* binding (13a), idioms (13b), resumption (20-24), extraposition (27), and adjectival modification (34), what we illustrate above is merely a correlation between R4-M and reconstruction, *whatever it is taken to be*. Of course, for those who believe the semantic approach does not aptly capture the empirical landscape, this chapter provides further support for syntactic reconstruction. We cannot do justice to this issue here, referring the reader to the exchange in Grosu & Krifka 2007, 2022; Bassi & Rasin 2018 for a recent reincarnation of the old debate, with a focus on intensional RCs.

list of candidates, but lists those he remembers on a piece of paper, which he hands to Alex, saying:

b. I listed here six judges we could face today on the other side of this door. **true** in (38a)

c. I listed here six judges on the other side of this door that we could face today. **false** in (38a)

d. I listed here six judges that are on the other side of this door who we could face today. **false** in (38a)

e. I listed here six individuals who we could face as judges on the other side of this door today. **true** in (38a)

This judgment is unexpected on our approach to R4-M. In each of (38c-38d), there is an additional element modifying the noun "*judges*" besides the modal-embedding RC that this noun heads: a PP in (38c) and a second RC in (38d). If these additional modifiers can be merged in the gap position of the intensional RC as schematized in (39)—and nothing in our system currently suggests that they cannot—then they should be able to reconstruct into that position, below the modal, and the sentence should have a true reading, meaning roughly what (38e) does.

(39) [D [[$_{\text{NP}}$ N [**modifier**]][**RC** … modal … [$_{\text{NP}}$ N [**modifier**]]]]]

A response to this challenge might consist of positing a syntactic principle that rules out structures like (39). Perhaps, for instance, structures like (39) are either ruled out, or heavily dispreferred relative to an alternative structure, in which the additional adjuncts modifying the head-NP do not merge RC-internally and raise with the head-NP. Rather, the head-NP merges RC-internally and raises on its own, and additional adjuncts are merged countercyclically, only to the copy of the NP in its derived position. This will prevent these modifiers from ever reconstructing into a position below the RC-internal modal, blocking R4-M.

(40) a. [D [[$_{\text{NP}}$ N][**RC** … modal … [$_{\text{NP}}$ N]]]] (raising of head-NP)

b. [D [[$_{\text{NP}}$ N [**modifier**]][**RC** … modal … [$_{\text{NP}}$ N]]]] (late-merge)

If this account is to be pursued, at least two questions must be addressed (though, alas, we are not currently in a position to answer either of them). One is empirical. Is there a *bias* against derivations like (39), or a *constraint* that categorically rules them out? While the sentences in (38c-38d) certainly

seem categorically false in their respective exclusively-R4-M-verifying scenario, we can come up with cases that are not as clear. For instance, consider (41b) in the exclusively-R4-M-verifying scenario in (41a).

(41) a. **Context:** A linguistics department is hiring a new faculty member. Alex, a faculty-member who is not part of the search committee, has particular preferences: She would like to hire a graduate of the University of California. After forming a long-list consisting of ten candidates, all of whom are currently finishing up their graduate studies, the head of the committee reaches out to Alex, saying: You'll be happy to hear that...

b. ...there are four professors with UC dissertations we could be welcoming to our department in the fall.

The judgments we have collected regarding (41) vary: almost all speakers we have consulted with found it harder to accept than our simpler examples of R4-M, where the head-NP is bare, but quite a few did. So the question is what sets the adjunct in (41b) apart from the ones in (38c-38d), and more generally, adjuncts that do not categorically block R4-M from those that do.

The second question is theoretical. If indeed there is (at the very least) a strong preference for derivations like (40) over (39), why should this be the case? One conjecture is that the tendency to merge things where we hear them is intended to ease the job of the linearization algorithm, which faces less choice points when linearizing a structure like (40) than it does given a structure like (39); only in the latter structure are there two copies of a modifier that needs to be linearized in just one position. We leave further discussion for future research.

2.7 Conclusion

Semantic orthodoxy has it that the quantificational scope of a determiner is not fully divorced from the intensional status of its restrictor. Szabó (2010, 2011) resurrects Fodor's (1970) old claim that it is, introducing a particular kind of intensional sentence judged true in a scenario that verifies it only if some determiner it contains is taken to quantify unrestrictedly, with the NP "restrictor" (a misnomer on this approach) evaluated in the domain of an intensional operator in the determiner's scope. Szabó uses this data, exemplifying a reading that we call here R4, to motivate an unorthodox "bare quantification" strategy for natural language, which splits determiners from their restrictors at LF, allowing them to quantify unrestrictedly over the entire domain of

quantification. Surprisingly, Szabó's data has not received much attention in the linguistic literature, despite its potential far-reaching implications for mainstream theories of intensionality and scope, which rule out R4.

We attempt to fill the gap by initiating a defense of orthodoxy, so to speak. First, we challenge the leap from the data to the conclusion by showing that bare quantification not only overgenerates unattested R4s, but also undergenerates a particular subclass of cases involving modal operators that Szabó offers as an argument in its favor. We then show that the undergenerated cases can be derived from a construction already independently argued to be available in natural language, of reconstructed intensional RCs, in which quantification is not bare, but restricted. The reconstruction account makes testable predictions that are indeed corroborated.

The scope of the challenge posed by Szabó is thus diminished, but the challenge itself is not fully defused. This is because—as mentioned at the outset—not *all* examples offered in favor of bare quantification are amenable to the reduction we defend for the modal cases. To see this, consider (42-43).

(42) a. **Context:** Under questioning by the police, Alex is presented with photos of her neighbors, and identifies those she thinks are thieves. She does not count her allegations, but an officer does, and reports to a colleague:

b. Alex believes that eleven thieves live in her building.

c. There are eleven individuals in the actual world such that Alex believes each of them to be thieves living in her building.

(adapted from Szabó 2010: ex. 7)

(43) a. **Context:** We are on a guided tour of what was once a prestigious camp for teenagers that had shut down years ago. The camp catered to teenagers educated in affluent private elementary schools. Every once in a while it also accepted students from the public school system, though never more than a handful at any given point in its existence. The tour guide says: this camp served some of the richest families in the state, but...

b. There were (also) fifteen high-schoolers/teenagers here who attended a public elementary school.

c. There are fifteen individuals, such that for each there is a past time interval during which they were high-schoolers/teenagers and another past time interval during which they attended a public elementary school.

(our example, modeled after Szabó 2011: ex. 28)

In (42), since Alex did not count her allegations, she does not believe *de dicto* that eleven thieves live in her building, and since the officer need not believe Alex's allegations, (42b) is not a statement regarding Alex's beliefs about eleven *actual* thieves, ruling out its *de re* and *third* readings. The sentence is thus true only if it expresses what the R4 paraphrase in (42c) does. In (43), it is matrix past tense that creates the intensional environment relative to which R4 is observed: In the context of (43a), the tour-guide does not intend to convey the *de dicto* interpretation of (43b), on which there is a past interval t such that fifteen high-schoolers *at* t previously attended public elementary schools. Nor is she saying anything about fifteen *utterance-time* high-schoolers, given that the camp shut down and its attendees are no longer in their teens, thus ruling out the *de re* and *third* readings. Rather, she means to convey what (43c) does; i.e., the R4 of (43b).

But (42b) contains no RC at all, and so the reconstruction account we propose for the modal cases is simply irrelevant. And while (43b) does contain relativization, its ability to convey the R4 paraphrase in (43c) cannot be due to reconstruction: Since one cannot be a teenager and attend an elementary school at the same time, reconstruction of the head-NP in (43b) results in interpreting it contemporaneously with a predicate that is temporally-incompatible with it, in the scope of embedded past tense. Alternative mechanisms are thus required to derive R4 in these examples.

Whatever these mechanisms turn out to be, our argument from overgeneration against the bare quantification strategy clearly suggests that they must be non-trivially restricted by—one would hope—the particular features of the intensional environments involved. In fact, recent proposals in the literature already point in exactly that direction. In particular, it has been suggested that the R4 of belief reports like (42b) be derived as particular instances of another phenomenon, first observed by Fauconnier (1984), and termed *revisionist belief* by Blumberg & Lederman (2021), in which an agent is truthfully ascribed belief in a proposition p despite their belief state failing to entail p, as long as p is entailed by a revision of their beliefs in light of the reporter's presuppositions.[25] Furthermore, in the next chapter we offer an account of the temporal cases that does not appeal to bare quantification. Here, we hoped to have convinced the reader with the argument above that one is better off attempting to derive different instances of R4 from distinct construction-specific mechanisms, than to unify them under a single unrestricted one.

[25] Cf. Benbaji-Elhadad 2023. Mayr & Schmitt (2026a) defend this approach by illustrating that R4 relative to attitudes seems subject to the same constraints governing the distribution of revisionist belief. However, in Benbaji-Elhadad 2026a, we point to some potential challenges their argument faces.

Chapter 3

Worlds, times, and the question of *ontological symmetry*

3.1 Introduction

It is standard to assume in model-theoretic semantics that declarative utterances express *propositions*, which are functions from worlds and times to truth values; e.g., the sentence *Mary arrived* denotes a proposition that holds of a possible world w and a time interval t iff Mary arrived at time t in w. There is a long-standing debate in the literature on modality and tense concerning the formalization of the world and time dependence of utterances, and in particular, concerning the question of whether worlds and times should be represented as pronouns in the object language just like pronouns that range over individuals. However, it is usually either implicitly assumed or explicitly asserted that regardless of one's position on that question, there should be no distinction between worlds and times; i.e., that the same machinery—*whatever it may be*—should capture the world and time dependence of natural language utterances (cf. van Benthem 1977; Cresswell 1990/2012; Schlenker 2006; von Fintel & Heim 2011). This thesis—which we call, following Schlenker (2006), *ontological symmetry*—is motivated by a large body of work, going back at least to Partee 1973, observing what appears to be "pervasive" analogous behavior of worlds and times in natural language (Schlenker 2006: 504).[1]

This chapter presents a new *dis*analogy between worlds and times. The

[1]Note that the thesis that Schlenker (2006) terms *Ontological Symmetry* is actually stronger than the way I will use this term: His thesis states not only that worlds and times should have parallel representation, but also that they should be represented as object language variables, just like individuals.

presentation consists of two parts: First, building on recent work by Schmitt (2023), I show that the way worlds are introduced into the semantic derivation must be strictly constrained to avoid overgeneration of a certain unattested reading of sentences containing modal operators. I then show that the same reading that is unattested with modal operators *is* attested with temporal ones, and furthermore, that it is attested exactly because the way time intervals are introduced into the semantic derivation is not subject to the constraint we must posit to prevent overgeneration in the modal case. Thus, worlds and times must be subject to distinct constraints. The rest of this introduction is dedicated to an overview of these subparts of the chapter.

3.1.1 Overview of chapter

Part I: Worlds

At the heart of this section are two assumptions that are quite common in the literature. First, that predicates of all kinds can be true of pluralities in virtue of being true of their atoms. For instance, in a scenario in which Jane arrived and Mary arrived, one can truthfully say that *Mary and Jane arrived*. Similarly, if it is the case that Jane fed Fido and that Mary fed Bob, then the sentence *Jane and Mary fed Fido and Bob* seems true. Thus, our semantic theory must posit a mechanism capable of accommodating such inferences from truths about individuals to truth about pluralities thereof, often called *cumulative inferences*. The second assumption is that intensional operators, like modal adverbs and attitude predicates, denote functions that take as their input what I will call *modal intensions*; namely, functions on the domain of possible worlds.[2] Thus, our semantic theory must posit a mechanism capable of introducing modal intensions into a semantic derivation when so required by the lexical items employed in that derivation.

Recently, Schmitt (2023) observed that possible worlds pattern differently from entities of all other types in environments that are sensitive to the plural part-structure of their sub-constituents: While pluralities of all types seem able to give rise to cumulative inferences, expressions previously argued to denote pluralities of worlds in particular seem unable to do so. Schmitt concludes that no natural language expressions denote world pluralities; a conclusion

[2]The objects I call here *modal intensions* are often simply called *intensions* in the modality literature, especially in work that is concerned with modals and attitude predicates, but not with tense. However, since I will be comparing the behavior of modal operators with that of tense operators, it will be useful to distinguish functions on the domain of worlds—i.e., modal intensions—from functions on the domain of time intervals, which I will call *temporal intensions*.

that contradicts a popular, and quite entrenched position in the literature, which attempts to derive certain inferences of intensional sentences from the assumption that certain linguistic expressions in these sentences denote such pluralities (e.g., Schlenker 2004; Križ 2019; Agha & Jeretič 2022).

This section argues that Schmitt's conclusion interacts in a surprising way with another theoretical claim that at first blush seems wholly unrelated to the nature of that conclusion. This is the claim that DPs cannot take wide quantificational scope relative to an intensional operator if their restrictor is evaluated *opaquely*, in the intensional environment created by the operator. The claim is old, quite broadly accepted—see, for instance, Ioup 1975; Keshet 2008, 2010b; von Fintel & Heim 2011; Keshet & Schwarz 2019; Benbaji 2021; Benbaji-Elhadad 2026b for arguments in favor of it, and Demirok 2019; Elliott 2023; Mirrazi 2024 for recent works that take it for granted—and is regularly justified by pointing out that certain sentences seem *false* in scenarios which, had wide quantificational scope and opaque evaluation gone hand in hand, would render these sentences *true* on a wide scope opaque construal.[3]

I show that this widely-accepted generalization is only predicted to hold if one accepts Schmitt's (2023) surprising conclusion about the absence of world pluralities: If the mechanism we posit to derive cumulative inferences is allowed to apply to modal intensions in the course of the semantic derivation this will render intensions true of pluralities of *worlds* by virtue of being true of the atomic members of these pluralities. Certain cumulative inferences with pluralities of worlds are in turn *entailed* by those readings that would be derived from the scopal configuration generally assumed not to exist. And, of course, any scenarios that verifies those latter readings also verify anything they entail. Thus, without a ban on cumulative readings with worlds, we wrongly predict certain sentences to be judged true, in scenarios that as a matter of fact render them false.

More specifically, I will consider a sentence like (1), uttered in a scenario in which only one victim is trapped under the rubble, but two particular individuals could each be that victim. Imagine, for instance, that a building collapsed with just one security guard inside it, but that rescue forces are unsure which of two possible guards was on duty during the collapse. The example is judged to be *false* in such a scenario. This lends support to the teachings of orthodoxy, of course, according to which the DP "*two victims*" cannot have its numeral

[3]The position is not *universally* accepted: Fodor (1970) famously argued that DPs can indeed scope above an intensional operators that determine the evaluation domain of their restrictors; a position that has been recently revived by Szabó (2010, 2011). In the previous chapter, and in Benbaji 2021; Benbaji-Elhadad 2026b, I defend the common view against Szabó's counter examples, which I therefore set aside for the purpose of the current chapter.

"*two*" scope above the modal "*might*" while its adjacent NP "*victims*" is evaluated in the epistemically accessible worlds quantified over by the modal. If it could have had that scopal configuration, we would expect the sentence to have a reading paraphrasable as in (2), which is *true* in the above scenario.

(1) Two victims might be trapped under the rubble.

(2) Two individuals (each) might be a victim trapped under the rubble.

I will show that on the common assumption that particular lexical elements in (1) denote modal intensions, we obtain a reading for the sentence that is equivalent to (2), once these intensions interact with standard mechanisms that derive cumulative inferences.

The point I will make can be summarized in a fairly non-technical way as follows: Cumulation mechanisms derive a true reading for a sentence like "*Two girls fed two cats*", even if there are no two girls who each fed two cats, as long as some girl fed some cat, and some other girl fed some other cat. More generally, these mechanisms allow a relation (like the one denoted by "*fed*" above) to hold between two plural indefinite quantifiers, if it holds between the members of the pluralities witnessing each indefinite in a way that no member of either witness is "left out" (Champollion, 2020).

If the predicate "*victim*" in (1) itself denotes a relation, true of an individual and a world if that individual is a victim in that world, cumulation should render that relation true of two *pluralities*, of individuals and worlds, just in case: (i) for each part of the individual plurality there is a world in the world plurality, in which it is a victim, and (ii) for each world in the world plurality there is a part of the individual plurality that is a victim in it. Then, as long as we allow the modal "*might*" to quantify existentially over pluralities of worlds, (1) should be rendered true by the existence of one world in which some individual is a victim, and another world in which some other individual is a victim. But this is precisely what goes on in the collapse scenario discussed above. Thus, (1) is predicted to have a reading that is equivalent to what would be derived by positing wide scope opaque scopal configurations, contrary to fact. To block the unattested reading, then, we must block the interaction between modal intensions and cumulation mechanisms.

The argument in this section thus provides a new motivation for Schmitt's (2023) conclusion: If we want to capture the empirical landscape *vis-a-vis* intensionality and scope correctly, we need to take for granted the conclusion that Schmitt motivates independently. However, the argument also goes a step further. Schmitt's argument examines *particular* expressions previously claimed to denote world pluralities—e.g., *if*-clauses (Schlenker, 2004), doxastic attitudes (Križ, 2019)—and shows that the world plurality hypothesis

overgenerates unattested cumulative readings of sentences containing these expressions. But Schmitt also notes that proponents of the world plurality hypothesis might attribute the absence of such cumulative readings to a stipulated quirk in the syntax or lexical semantics of the relevant expressions. While this approach is conceptually problematic—it renders world pluralities effectively undiagnoseable by appealing to cumulativity (and, perhaps, the world-plurality hypothesis unfalsifiable)—it has not yet been shown to be empirically inadequate.

I show that, indeed, the problem Schmitt identifies cannot be handled in this way, as it does not only concern those particular modal expressions. Rather, we can apply the logic of Schmitt's argument to illustrate that the mere presence of elements that denote modal intensions in the structural representation of a sentence, when paired with the pluralizing mechanisms that derive cumulative inferences, leads to readings long thought to be unattested.[4]

In the more speculative part of this section, I will explore whether we can block those unattested readings once we adopt a particular approach to intensionality—*The Scope Theory of Intensionality*, dating back to Montague (1970, 1973). As a preview, the conclusion I will arrive at is that the scope theory's design and basic principles do not by themselves block the unattested reading, but, together with certain general principles pertaining to cumulativity we can correctly capture the empirical landscape under this theory. The discussion is not conclusive, in that it does not offer a comprehensive defense of these general principles or address the numerous worries that pairing them with the scope theory raises. Rather, it lists some reasons why pursuing this approach, and addressing the worries to be discussed, is a promising endeavor.

[4]Since the inception of possible worlds semantics for natural language, there has been a steady stream of arguments against its representation of propositions, information states, and content more generally as sets of worlds. These arguments all suggest that possible-worldly representations are too coarse-grained; they do not furnish us with the tools to distinguish between sentences expressing equivalent propositions, which—as a matter of fact—do pattern differently in certain linguistic contexts. Many have, consequently, gravitated towards a view of content whose primitive elements are not worlds, but rather finer-grained entities like situations, states of affairs, or truthmakers (for some instructive examples, see, *amo*, Kratzer 2007/2023; Yablo 2014; Fine 2017; Moltmann 2020a; Champollion 2025). Semantic formalisms that make use of such finer-grained representations are all more expressive than possible worlds semantics, given their ability to draw distinctions that are obscured by possible-worldly representations. This chapter swims against that popular current: The argument it presents illustrates that possible worlds semantics is already itself *too expressive*, giving rise to unattested readings simply in virtue of the fact that it construes the denotations of certain lexical items as functions on worlds. As far as I can tell, the argument should extend, *mutatis mutandis*, to any theory in which functions on any finer-grained entities constitute the denotations of any linguistic material.

Part II: Times

Next, I argue that regardless of whether one accepts my defense of the scope theory, the constraints we must adopt to block the unattested readings of sentences like (1)—whatever they may be—are *too* restrictive if we think that functions over time intervals, which I will call *temporal intensions*, are introduced into the semantic derivation via the same mechanism that introduces modal intensions. While avoiding overgeneration demands that modal intensions be introduced by a mechanism that blocks them from interacting with cumulation, temporal intensions should be allowed to freely interact with cumulation to account for the range of attested readings when tense is involved, suggesting that the two kinds of intensions should not be introduced into the derivation via the same mechanism.

The argument consists of two claims. The first is empirical: Unlike the sentence in (1), certain temporal sentences *do* seem to have the equivalent of a wide scope opaque reading of a DP they contain relative to a tense operator. This claim is due to Szabó (2010, 2011),[5] who exemplifies it with sentences like (3).[6] This sentence seems to have a reading paraphrasable as in (4), where the quantificational force of "*eleven*" is above matrix past tense, but the NP "teenagers" holds of the individuals being quantified over in the past, but not at present.

(3) There were eleven teenagers who attended public kindergarten.

(4) There were eleven individuals such that for each there is a past time interval during which they were a teenager, and a time before that, during which they attended a public kindergarten.

The second claim is theoretical: I argue that (at least some cases of) the reading relative to tense *must* be derived by applying standard cumulation mechanisms to temporal intensions; i.e., that the kind of interaction that must be blocked between cumulation and modal intensions cannot be blocked for temporal intensions. Defending this claim will require that we rule out potential alternative derivations of the apparent wide scope opaque reading, and a part of the discussion of the temporal cases is devoted to that task.

[5]Note that Szabó makes an expansive claim that intensional sentences cutting across the primary intensional contexts—attitudes, modals, and tense—give rise to wide scope opaque readings. Example (1) shows that this claim certainly does not hold of *all* modal sentences, and in the previous chapter I explore some of the modal cases provided by Szabó, illustrating that they can be derived in a way that does not require cumulation of modal intensions and are thus, in a sense, orthogonal to the present chapter.

[6]This sentence is my version of Szabó's example, intended to avoid potential confounds to be discussed later on.

Part III: The question of *Ontological Symmetry*

Our findings in the two parts of the chapter, jointly, have potential implications for the way we represent world and time dependence in our semantic formalism. First, if one accepts our more speculative proposal that the derivation of the unattested readings in the modal case should be blocked by designing the semantic formalism in a particular way—namely, by adopting the scope theory of intensionality with respect to the representation of worlds—then we must make sure that worlds and times are handled by different formalisms. Otherwise, our formalism will block the reading for tense as well, contrary to fact. But even if one attempts to handle worlds and times within the same formal framework (rejecting the ideas concerning STI), the constraints on evaluation indices would have to distinguish worlds and times in order to account for the absence of the reading. The chapter concludes by underscoring the challenge of this finding for the thesis of ontological symmetry.

3.1.2 Roadmap

Section 3.2 presents (parts of) Schmitt's (2023) argument for the ban on pluralities of worlds, setting the stage for the new argument. Section 3.3 then presents the standard picture regarding the unavailability of wide scope opaque readings, and Section 3.4 illustrates how different theories of intensionality attempt to block these readings. Section 3.5 shows how, once these theories are augmented with a cumulation mechanism they actually fail to block the unattested readings. Section 3.6 speculates on how the unseemly interaction between cumulation and intensionality can be blocked under a particular view of intensionality. This concludes the first part of the argument, which pertains to worlds and modal intensions. The second part, consisting of Section 3.7, illustrates that the readings unattested for modal sentences are in fact attested for temporal ones, can be derived as cumulative inferences with pluralities of times, and furthermore *should* be derived in this way. Section 3.8 concludes by weighing the implication of the findings in the chapter for our theory of intensionality, and considering some open challenges.

3.2 The ban on cumulativity with pluralities of worlds

Plural predication gives rise to certain *cumulative* inferences from truths about individuals to truths about pluralities thereof. The truth of the sentences in

(5a-5c), for instance, in which the one-place predicate "*slept*" is asserted to hold of several individuals separately, entails the truth of the sentence in (5d), where the predicate is applied to a plurality of said individuals (presumably denoted by the conjunction; Link 1983). Similarly, the truth of the three sentences in (6a-6c)—where the two-place predicate "*talk to*" is asserted to hold of several pairs of individuals—suffices for us to conclude that the predicate also holds of the pair consisting of the pluralities of these individuals, as indicated by the truth of (6d) given (6a-6c).

(5) a. Sue slept.
b. Jane slept.
c. Mary slept.
d. ∴ Mary, Jane and Sue slept.

(6) a. Student 1 talked to Professor 1.
b. Student 1 talked to Professor 2.
c. Student 2 talked to Professor 3.
d. ∴ The two students talked to the three professors.

More abstractly, we can define a cumulative inference as in (7), for predicates of any arity, though we will only be concerned with one- and two-place ones.

(7) For any n-place predicate P^n, and any m n-tuples of arguments,
$< x_1^1, x_2^1, \ldots, x_n^1 >, < x_1^2, x_2^2, \ldots, x_n^2 >, \ldots, < x_1^m, x_2^m, \ldots, x_n^m >$
(where the lower index on every variable represents its position within its tuple, and the upper index represents the tuple of which it is a part), the following is a cumulative inference
$< x_1^1, \ldots, x_n^1 > \in P^n \wedge < x_1^2, \ldots, x_n^2 > \in P^n \wedge \ldots \wedge < x_1^m, \ldots, x_n^m > \in P^n \Rightarrow$
$< x_1^1 \oplus \ldots \oplus x_1^m, x_2^1 \oplus \ldots \oplus x_2^m, \ldots, x_n^1 \oplus \ldots \oplus n_n^m > \in P^n$
(where $\oplus$, here and throughout, is the sum-formation operation, returning for any two individuals, the individual that is their mereological sum)

Schmitt's (2023) argument against the existence of world-plurality denoting expressions uses the presence of cumulative inferences to diagnose for plural denotations in a sentence. It goes roughly as follows: First, it is observed that conjunctions of propositions may partake in cumulative relations with other plurality-denoting expressions. Yet, no expressions previously argued to denote pluralities of worlds partake in cumulative relations with propositional conjunctions. Thus, if cumulativity is a reliable diagnostic for plurality (cf. Schmitt 2019), an analysis of these constructions in terms of world pluralities seems to be on the wrong track.

To see that conjunctions of propositions "count" as plural expressions for the purpose of deriving a cumulative reading, consider (8), where a plurality consisting of John and Bill is truthfully ascribed belief in a conjunction of propositions, even though neither John nor Bill can truthfully be ascribed belief in the conjunction on his own: In the given context, each individual only believes one of the conjoined propositions.

(8) a. **Context:** Sam owns a construction company. Bill and John are her clients. John believes Sam is from Texas, and has no opinion on whether she is rich, while Bill believes she is rich but does not know where she is from.

b. John and Bill believe that Sam is from Texas and that she is rich. (Marty, 2019)

Pasternak (2018) attributes the felicity of such ascriptions of "joint belief" to the fact that the beliefs of John and Bill are mutually compatible, and thus, the conjunction of their belief states entails the conjunction in the prejacent of "*believe*" in (8b). However, Marty (2019) shows that joint belief attribution subscribes to the condition on cumulativity, mentioned earlier (and made evident in (7)), according to which a relation holds cumulatively between two pluralities only if it does not "leave out" any member of either. The condition, spelled-out specifically for joint belief in (9), plays a role once the context is manipulated as follows: Suppose John believes Sam is from Texas, but Bill only believes that if Sam is from Texas, she is rich. In this context, (8b) no longer seems true, despite the fact that the conjunction of John and Bill's belief states still entails the conjunction in the complement of the attitude verb. What is different now is that the modified context violates the condition in (9), and as Marty notes, this is at least suggestive that "joint belief" reduces to a cumulative belief-relation holding between a plural subject and a conjunction of propositions.

(9) An individual plurality $x_1 \oplus \ldots \oplus x_i$ can be ascribed joint belief in a conjunction $p_1 \wedge \ldots \wedge p_j$ only if the *belief*-relation holds *cumulatively* between the plurality and the conjunction: i.e., only if each atom in $x_1 \oplus \ldots \oplus x_i$ believes at least one conjunct in $p_1 \wedge \ldots \wedge p_j$, and each conjunct in $p_1 \wedge \ldots \wedge p_j$ is believed by at least one atom in $x_1 \oplus \ldots \oplus x_i$.

Schmitt (2023) illustrates that in modal constructions previously analyzed as involving pluralities of worlds, the hypothesized world pluralities fail to relate cumulatively to conjunctions of propositions. Below, we reproduce Schmitt's illustration that *if*-clauses, analyzed as world pluralities by Schlenker

(2004), Gajewski (2005), and Križ (2015), do not partake in a cumulative relation with a propositional conjunction. We then provide a parallel illustration for weak necessity modals, also recently analyzed as plurality-denoting expressions by Agha & Jeretič (2022).

On the world-plurality analysis of *if*-clauses, "*if*" is assigned, essentially, the semantics of a definite article: Given a proposition p and world argument w, it returns *the* plurality of p-worlds maximally similar to w, given a similarity ordering. The conditional as a whole then predicates its consequent proposition on the world plurality denoted by its antecedent. This is spelled-out in (10).

(10) a. $[\![if_w]\!] = \lambda p.\ \bigoplus \text{CLOSEST}(w)(p)$ (where, $\forall w, p : \bigoplus \text{CLOSEST}(w)(p)$ is the plurality of p-worlds most similar to w)

b. $[\![[if_w\ \phi]\ [\psi]]\!] = [\![\psi]\!]([\![if_w]\!]([\![\phi]\!])) = [\![\psi]\!](\bigoplus \text{CLOSEST}(w)([\![\phi]\!]))$

c. $[\![$*If Abe wins, Ron will be happy*$]\!]^w = 1$ *iff* Ron is happy in the (maximal) world plurality in which Abe wins and which is otherwise maximally similar to w

A conditional thus asserts that its consequent stands in a particular relation to the plurality in its antecedent; namely, the *be-true-of* or *hold-of* relation. Schmitt shows that this relation cannot hold cumulatively via conditionals with a conjunctive consequent. If propositional conjunctions "count" as pluralities, such conditionals are predicted to behave as if their consequent is interpreted *disjunctively*. This is because the *hold-of* relation should hold cumulatively between a conjunction and a plurality of worlds, even if each proposition in the conjunction does not hold of each world in the plurality, as long as each world in the plurality makes true *some* proposition in the conjunction (and each conjunct is true of some world). It is easy to see that such a disjunctive interpretation is unavailable for, e.g., (11b), despite the fact that the scenario in (11a) renders the utterance's cumulative paraphrase in (11c) true.

(11) a. **Context:** If Abe's company does not go bankrupt, he will hire another employee; either Sasha or Bert, but certainly not both.

b. If Abe's company does not go bankrupt, he will hire Sasha and he will hire Bert.

c. In each of the closest worlds where Abe's company does not go bankrupt, Abe hires Sasha, or he hires Bert.

The argument generalizes to any construction argued to ascribe the *hold-of* relation to a world plurality and a conjunction of propositions. Since the relation is expected to hold *cumulatively* between two pluralities, we predict the

weak disjunctive interpretation to be available. If it is not, we should question the assumption that a world plurality is involved to begin with. For instance, we can apply the argument to weak necessity modals. On their analysis in Agha & Jeretič (2022), they too are definite determiners. Simplifying significantly, the modal "*should*" takes a domain D and a world w, defined as a set of worlds that are "best" given some contextually supplied ranking among the worlds accessible from w, and returns their maximal plurality. The modal's propositional complement is then predicated to hold of this plurality; cf. (12). And with necessity modals, too, the weak cumulative reading does not seem to arise, as illustrated by the fact that the context in (13a) seems to falsify (13b), despite verifying its cumulative paraphrase (13c).

(12) a. $[\![\text{should}_w\ D]\!] = \bigoplus D$

b. $[\![[[\text{should}_w\ D]\ \phi]]\!] = [\![\phi]\!]([\![\text{should}_w\ D]\!]) = [\![\phi]\!](\bigoplus D)$

(13) a. **Context:** The syllabus of an advanced semantics class requires that students read a paper or solve an exercise, but not both.

b. According to the syllabus, you should read a paper and solve an exercise.

c. In each "best" world given the syllabus, Abe reads a paper or solves an exercise.

From these observations (and others, which I omit here for conciseness), Schmitt (2023) concludes that overt expressions previously argued to form pluralities of worlds do not, in fact, do so.[7] However, as Schmitt notes, proponents of the world plurality hypothesis have some potential workarounds. For instance, they can stipulate that the absence of cumulative readings in (11) and (13) is due to a quirk in the syntax or lexical semantics of conditional operators and necessity modals, which prevents the grammatical devices in charge of deriving cumulativity from applying. Perhaps these elements denote world pluralities that are "inherently distributive"—either by mandatorily combining with a distributivity operator or by encoding distributivity into their lexical

[7] The world plurality analysis in, e.g., (10) and (12) is motivated primarily by the observation that *if*-clauses and weak necessity modals give rise to *homogeneity* and *non-maximality* effects, which are attested also in the presence of expressions denoting *individual* pluralities (cf. Gajewski 2005). If Schmitt is right to reject the world plurality analyses, these shared interpretative effects require an alternative explanation. I set this issue aside for the purposes of the current chapter, but note that, as Schmitt already points out (citing Burnett 2016) homogeneity and non-maximality arise even in the absence of plurality, particularly in certain degree constructions. It is possible, then, that these effects are correlated with an independent property shared by certain plural and non-plural expressions, and are thus unreliable diagnostics for plurality, in contrast to cumulativity.

entries—thus requiring the world pluralities they denote to render true *every* proposition in the conjunction they combine with? In that case, the world pluralities involved are not expected to partake in cumulative relations in the first place. Alternatively, they can reject the assumption that conjunctions of propositions "count" as pluralities, and instead pursue Pasternak's (2018) analysis of examples like (8) as instances of "joint belief" rather than as instances in which a cumulative *belief*-relation holds between a plurality of individuals and a conjunction of propositions.[8]

Schmitt lists some conceptual arguments against this approach, noting that it would make plurality in the world domain undiagnosable and therefore, the world plurality hypothesis unfalsifiable. Furthermore, if a world plurality analysis is appealed to in the first place to model certain interpretative parallels to individual plurality, introducing inherent asymmetry raises non-trivial questions; in particular, why should only *some* properties that characterize individual plurality carry over to the world domain if not all of them do?

But even setting aside these conceptual reservations, the argument to be presented next illustrates that stipulations of this sort are also *empirically* inadequate. I will show that the scope of Schmitt's generalization—that worlds do not partake in cumulative readings—goes beyond the particular expressions she examines, and thus that deriving the absence of a cumulative reading in (11) or (13) by analyzing *if*-clauses and necessity modals in a particular way, or by rejecting Schmitt's assumption about propositional conjunctions, simply would not do. The argument takes as its starting point the observation that a certain scopal configuration seems systematically unavailable for DPs in intensional context, an observation that we turn to next.

3.3 On the non-existence of wide scope opaque readings

DPs in intensional environments may vary both in the scope they take relative to an intensional operator, and in whether their NP restrictors are evaluated *opaquely*, in the worlds quantified by that operator, or *transparently*, in the matrix world of evaluation. However, these two dimensions of variation are not independent: Wide quantificational scope for a DP blocks opaque evaluation of its restrictor.

[8]Of course, in that case, one would have to explain why such cases are sensitive to the condition on cumulative relations, which requires such relations to hold between two pluralities without leaving out any members of those pluralities.

This is often illustrated with the well-known sentence in (14), due to Fodor (1970). The sentence contains an existential DP "*an inexpensive coat*" and the attitude predicate "*want*", which we take to be a universal quantifier over possible worlds that satisfy Mary's desires (Hintikka, 1962).

(14) Mary wants to buy an inexpensive coat. (Fodor 1970)

(15) a. $\lambda w.\ \forall w' \in \text{DESIRE}_{Mary,w}\colon \exists x\colon \textit{cheap-coat}_{w'}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(de dicto)*

b. $\lambda w.\ \exists x\colon \textit{cheap-coat}_{w}(x) \wedge \forall w' \in \text{DESIRE}_{Mary,w}\colon \textit{buy}_{w'}(Mary, x)$ *(de re)*

c. $\lambda w.\ \forall w' \in \text{DESIRE}_{Mary,w}\colon \exists x\colon \textit{cheap-coat}_{w}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(third)*

d. $\lambda w.\ \exists x\colon \forall w' \in \text{DESIRE}_{Mary,w}\colon \textit{cheap-coat}_{w'}(x) \wedge \textit{buy}_{w'}(Mary, x)$ *(R4)*

The sentence has three readings: (i) A *de dicto* reading, true as long as the purchase of some inexpensive coat or other meets Mary's desires. (ii) A *de re* reading, verified by there being a *specific* inexpensive coat Mary wants to buy, possibly without knowing that it is inexpensive, or even a coat. And (iii) a *third* reading, verified by the existence of several objects such that buying either would satisfy Mary, which all happen to be inexpensive coats (again, possibly unbeknownst to Mary). The *de dicto* reading is often analyzed as involving narrow scope for the existential DP relative to the attitude with opaque evaluation of its restrictor (15a), the *de re* reading pairs wide scope with transparent evaluation (15b), and the third reading involves narrow scope with transparent evaluation (15c).

A reading that (14) does *not* seem to have, on the other hand, is the one formalized in (15d), which would be derived had the existential DP outscoped the attitude verb while its restrictor were evaluated opaquely, and has been called in the literature its *wide scope opaque reading* or its *fourth reading*. Going forward, I will adopt the latter term, henceforth **R4**, which is more theory neutral than the former. R4 is verified by the existence of a particular object that Mary wants to buy, thinking that it is inexpensive, even if that object is actually quite expensive. To illustrate the non-existence of R4, Keshet (2008) notes that the R4 of (14) is equivalent to (16a). But (16a) is not a possible interpretation of (14), as illustrated by the fact that when pronominal reference is used to force wide scope for the DP over the attitude verb, as in (16b), opaque evaluation of the term "*inexpensive*" is rendered unavailable.

(16) a. There's a coat that Mary wants to buy. She thinks it is inexpensive. But really, it is quite expensive.

b. Mary wants to buy an inexpensive coat.
#But really, it is quite expensive.

	Transparent	Opaque
Wide-scope (*specific*)	✓	✗
Narrow-scope (*non-specific*)	✓	✓

Table 3.1: DPs in intensional contexts

Table 3.1 thus represents the picture accepted by much of the semantic literature, according to which wide scope and opaque evaluation are incompatible with each other.[9] Indeed, contemporary discussions of intensionality and scope often treat the ability to capture the empirical landscape exactly as reflected in the table—i.e., blocking the derivation of R4, the reading that would be derived by combining wide scope and opaque evaluation—as a benchmark for an empirically adequate theory of intensionality (e.g., Demirok 2019; Elliott 2023; Mirrazi 2024). Next, I illustrate how two families of theories of intensionality are designed to block R4, focusing on *The Scope Theory of Intensionality* and its modern rendition in Keshet's (2008; 2010b) termed *Split Intensionality*, and on *The World Variable Framework* whose modern incarnation was developed by Percus (2000), building on a long tradition in modal logic (cf. among many others Tichý 1971; Gallin 1975; van Benthem 1977; Groenendijk & Stokhof 1982; Zimmermann 1989; Cresswell 1990/2012).

There are two reasons to focus on these theories in particular: First, they are very popular, with the world variable framework in particular constituting the go-to formal framework in which intensional constructions are analyzed (Yanovich 2015: 66), and with Keshet's split intensionality system treated as "the state of the art scope theory" (Elliott 2023: 334). More importantly, for the purposes of this chapter, the two approaches represent here two *families* of approaches, differing in their assumptions about the range of elements that object-language expressions may denote, and my eventual goal is to show that regardless of such assumptions, both kinds of approaches suffer from an overlooked overgeneration problem.

[9]My presentation in this section is somewhat anachronistic: In fact, Fodor (1970) originally claimed that R4 is attested, and offered (14) as an illustration of a sentence that does have this reading. Subsequent research rejected Fodor's original judgment (Ioup, 1975; Keshet, 2008, 2010b; von Fintel & Heim, 2011; Keshet & Schwarz, 2019), with Keshet 2008 offering the data above as an argument in favor of this rejection. Szabó (2010, 2011) argues that the rejection of Fodor's main claim, that R4 exists, was too hasty, presenting both new data purportedly illustrating the existence of R4, and an argument that this data is indicative of logical forms in which an intensional operator scopes between the determiner and restrictor of a single DP. Cf. footnote 5.

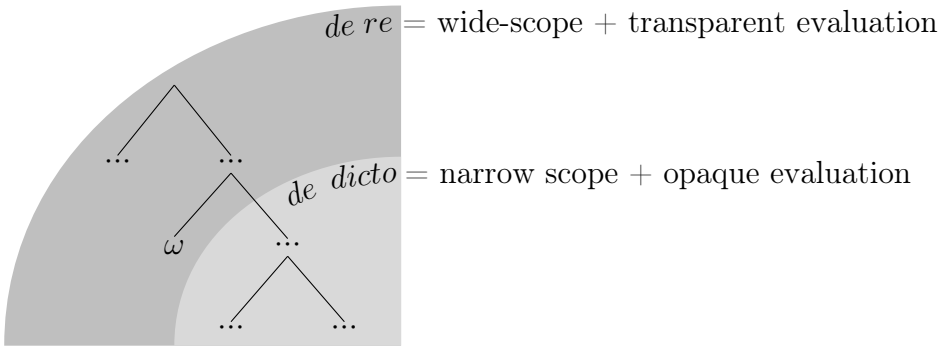


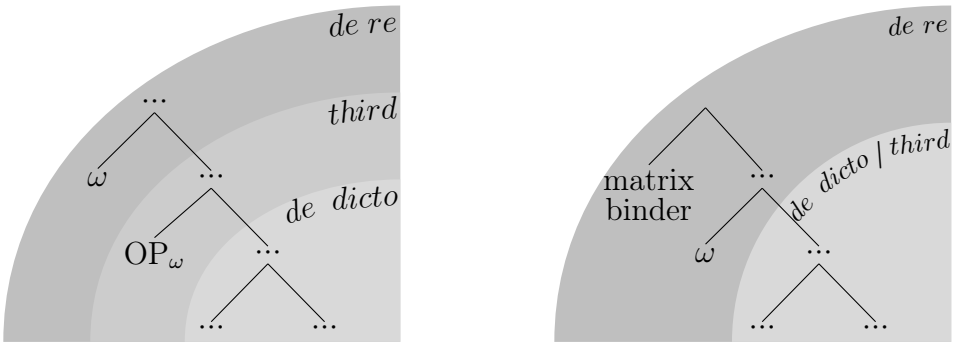
Figure 3.1: STI: wide-scope entails transparent evaluation; narrow-scope entails opaque evaluation

3.4 Blocking the fourth reading

The two systems to be considered here were both developed in response to the shortcomings of the classical *Scope Theory of Intensionality* (henceforth, **STI**; Montague 1970, 1973; Ladusaw 1977).[10] STI maintains that the intensional status of a world-dependent element is determined by the nearest intensional operator ω above it at LF; i.e., that every world-dependent element in the c-command domain of ω is interpreted opaquely relative to ω. Paired with a syntactic approach to scope, STI predicts a biconditional dependence between quantificational scope and intensional status: as shown in Figure 3.1, any element under ω will receive narrow-scope and opaque evaluation relative to ω, and any element taking scope above ω will receive transparent evaluation. As desired, biconditional dependence (seemingly) rules out R4, which requires wide scope and opaque evaluation. However, it also rules out the narrow-scope transparent *third* readings. STI, then, fails to capture the empirical landscape spelled-out in Table 3.1; the feature of the theory in charge of blocking unattested cases of R4—namely, the biconditional dependence of scope and intensional status—also leads to the undergeneration of attested ones.

Keshet (2008, 2010b) modifies STI by partially divorcing scope from intensional status. On his modified version, being below ω at LF entails narrow-scope, but is not sufficient for opacity. DPs are opaque only if they reside below *some other operator*, call it OP_ω, itself situated below ω at LF. In the resulting system of *split intensionality*, there is a region under ω where elements receive

[10]The discussion here, of STI and its successors, closely follows Keshet & Schwarz 2019, and the figures below are reproduced from Benbaji-Elhadad 2026b. For simplicity, I ignore times, exemplifying the theories solely with respect to world quantifiers.



(a) Split intensionality (b) World variables

Figure 3.2: Modifications of STI: wide-scope entails transparent evaluation while narrow-scope is compatible with transparent or opaque evaluation

transparent evaluation; namely, the region between ω and OP_ω. As illustrated in Figure 3.2-a, as in the case of STI, syntactic position still fully determines a DP's intensional status in the modified system, but STI's biconditional dependence becomes a conditional one: wide-scope entails transparency, but not vice versa.

The same conditional dependence is the result of Percus's (2000) more radical departure from STI. In his system, world-dependent lexical items are functions whose input is a world-argument, saturated by silent object-language *world-variables* at LF. The restrictor of every DP is assumed to contain such a variable as its argument, which can be bound either by an intensional operator above it or by a binder posited at the matrix level, with the identity of the binder determining the intensional status of the DP. When a world-variable is bound by the matrix binder, the restrictor is transparent, and it is opaque when its variable is bound by the intensional operator above it. As shown in Figure 3.2-b, here, too, residing under ω does not determine a DP's intensional status, but being outside it does: ω cannot bind the world-pronoun of a DP above it, which must thus be evaluated transparently.

Both approaches generate the *third* reading by positing a mechanism that allows elements in the c-command domain of an intensional operator ω to escape opacity relative to ω. However, there is no mechanism in place that would allow an element outside the c-command of ω to nevertheless be opaque. Thus, R4 is still ruled out by both modifications of STI. Or is it?

3.5 Failing to block the fourth reading

Next, I illustrate that certain cases of R4 can still be derived within the standard theories of intensionality schematized above, despite those features of the theories intended to block them. As noted, these features ensure that a DP will not be able to occupy a position above an operator that in turn determines the intensional status of its restrictor. However, what I show next is that given some independently needed interpretative mechanisms, one can derive for certain sentences, under either theory, a reading that entails R4, even when those sentences are mapped to LFs in which the relevant DP is positioned *below* an intensional operator. We thus predict such sentences to be true in scenarios that are verified by R4, contrary to fact.

The sentences I have in mind contain two existential quantifiers: A plural indefinite DP, quantifying existentially over individual pluralities, and a possibility modal, quantifying existentially over possible worlds. One example, recall, is (17), which contains the epistemic modal "*might*" and the plural indefinite "*two victims*".

(17) Two victims might be trapped under the rubble.

The main empirical observation I will be concerned with consists of the fact that (17) does not seem to be verified by the existence of two individuals who could *each* be a victim trapped under the rubble, even when there are no two individuals who could *both* be victims trapped under the rubble. This is the state of affairs in the scenario in (18) which, indeed, does not seem to render (17) true.

(18) **Collapse scenario:** A building collapsed, with only the security guard on the night shift present inside. There are only two people who are ever on duty at night, but we are not sure which of them picked up the shift on the night of the collapse. Thus, for all we know, each of them *might* be a victim of the collapse that is trapped under the rubble, but we are certain that it is not the case that both are.

R4 of (17) is predicted to be *true* in this scenario, as it pairs wide quantificational scope for "*two victims*" relative to the modal with opaque evaluation of the restrictor, and is thus paraphrasable as follows: There are two individuals who could each be a victim trapped under the rubble.[11] Therefore, we must

[11] I introduce the distributivity operator "*each*" in the paraphrase, which indicates that the predicate in the scope of the plural indefinite quantifier must hold of every atomic individual in the pluralities being existentially quantified over. This is because any two existential quantifiers are scopally-commutative without distributivity; i.e., the expression

ensure that R4 is not derivable for (17).[12]

In the remainder of this section, I will show how we can derive for (17) precisely this reading, under either of the two approaches to intensionality presented above, augmented with a mechanism that derives *cumulative inferences*. To that end, I will adopt a working hypothesis regarding how cumulative inferences come about. I will then show how augmenting each of the theories of intensionality introduced in Section 3.4 with a cumulation mechanism wrongly predicts the R4 of a sentence like (17) to be attested.

However, since that discussion will naturally involve quite a bit of technicalities, let us first briefly formulate the essential points to be made below in non-technical terms. First, in both the world variable system and the system of split intensionality, the predicate "*victims*" can contribute to the semantic derivation a relation, true of an individual and a world if that individual is a victim in that world. I will apply a cumulation mechanism to that relation, rendering it true of two *pluralities*, of individuals and worlds, just in case: (i) for each part of the individual plurality there is a world in the world plurality, in which it is a victim, and (ii) for each world in the world plurality there is a part of the individual plurality that is a victim in it. Then, I will also use cumulation to allow the modal "*might*" to quantify existentially over pluralities of worlds. Together, this will render (17) true by the existence of one world in which some individual is a victim, and another world in which some other

---

obtained from scoping one above the other entails the expression obtained from reversing this scopal configuration, as long as the scope of the inner quantifier remains constant (for any scope ϕ, $[\exists x \exists y \phi(x, y)] \Leftrightarrow [\exists y \exists x \phi(x, y)]$). Therefore, we might expect readings that differ only in the scope they assign to one existential over another to be indistinguishable. In particular, the R4 of (17) without distributivity, according to which for two individuals there is a world in which they are (both) trapped under the rubble, seems to be equivalent to, and thus indistinguishable from, its *de dicto* reading, according to which there is a world in which two victims are trapped under the rubble. Whether or not commutativity should be assumed to hold when one of the existential quantifiers involved is a modal, as is the case here, is subject to some debate in the study of modal logic (see, for instance Williamson's (2013) discussion of the *Barcan formula* and its converse, due to Barcan (1946)), and the introduction of distributivity here allows us to side-step this debate altogether.

[12]We can reproduce Keshet's (2008) argument, illustrating that Fodor's (1970) sentence in (14) lacks a R4, for (17) as well. If the sentence had a reading equivalent to the R4 paraphrase, a pronoun in a succeeding sentence should be able to refer back to those individuals individuated by the wide-scope existential DP, who are potential but not actual victims. The contrast below, modeled after (16) illustrates that this is not the case.

(i) a. There are two individuals who could each be a victim trapped under the rubble. However, they are not both victims.

b. Two victims might be trapped under the rubble. #However, they are not both victims.

individual is a victim, which is, in turn, precisely what goes on the collapse scenario. Thus, (17) is predicted by both systems to be true in a scenario that exclusively verifies its R4, contrary to fact.

3.5.1 Assumptions about the cumulation mechanism

Recall that cumulative inferences are the inferences of the (d)-sentences below, drawn from the (a-c)-sentences. Given that we seem justified in drawing such inferences, we need a mechanism in place that will render the (d)-sentences true, in case the assumptions in (a-c) are.

(19) a. Sue slept.
b. Jane slept.
c. Mary slept.
d. ∴ Mary, Jane and Sue slept.

(20) a. Student 1 talked to Professor 1.
b. Student 1 talked to Professor 2.
c. Student 2 talked to Professor 3.
d. ∴ The two students talked to the three professors.

Let me mention two such mechanisms in particular. On one approach, the extensions of predicates are closed under sum-formation as part of their *lexical meaning*; e.g., the denotation of a verb like "*slept*" consists of the set of all individuals who slept, as well as of any individual formed by summing up any n sleepers (cf. Scha 1981; Krifka 1992; Landman 1996; Winter 2000; Kratzer 2003, 2008, *ao*). In other words, it is part of the lexically specified meaning of predicates that they give rise to cumulative inferences. This explains, for instance, why (19d) is true when (19a-19c) are; since the atomic individuals Sue, Jane and Mary are in the extension of "*slept*", then *ipso facto* so is the individual formed by summing them up.

On another approach, closure of predicate extensions under sum-formation is not a lexical property, but the result of applying cumulation operators to predicates at LF (cf. Link 1983; Krifka 1986; Sternefeld 1998; Beck & Sauerland 2000, *ao*). These include Link's (1983) "star-operator" $*$, which applies to one-place predicates, Krifka's (1986) extension of it to two-place ones, "double-star" $**$, and any further extensions to predicates of higher-arity (which we henceforth ignore for simplicity). Type-flexible versions of these operators are defined in (21a-21b), with the effect of the latter on the two-place predicate "*talk to*" exemplified in (22), illustrating why the cumulative inference in (20) goes through.

(21) a. $[\![*]\!] = \lambda\mathcal{F}_{\langle\sigma,t\rangle}\lambda x_{\sigma}.\ \exists\mathcal{F}'_{\langle\sigma,t\rangle} : [\forall x' : \mathcal{F}'(x') \rightarrow \mathcal{F}(x')] \ \wedge \ x = \bigoplus\{x \mid \mathcal{F}'(x)\}$

b. $[\![**]\!] = \lambda\mathcal{R}_{\langle\sigma,\langle\tau,t\rangle\rangle}\lambda x_{\sigma}\lambda y_{\tau}.\ \exists\mathcal{R}'_{\langle\sigma,\langle\tau,t\rangle\rangle} : [\forall x', y' : \mathcal{R}'(x', y') \rightarrow \mathcal{R}(x', y')] \wedge x = \bigoplus\{x \mid \exists y : \mathcal{R}'(x, y)\} \ \wedge \ y = \bigoplus\{y \mid \exists x : \mathcal{R}'(x, y)\}$

(22) a. [talk to] = {<Student 1,Prof 1>, <Student 1,Prof 2>, <Student 2,Prof 3>}

b. [**]([talk to]) = {<Student 1,Prof 1>, <Student 1,Prof 2>,
<Student 2,Prof 3>, <Student 1,Prof 1⊕Prof 2>,
<Student 1⊕Student 2,Prof 1⊕Prof 3>,
<Student 1⊕Student 2,Prof 2⊕Prof 3>,
<Student 1⊕Student 2,Prof 1⊕Prof 2⊕Prof 3>}

The operator approach *subsumes* the lexical one in that any cumulative inference derived by assuming predicates are "born" with their arguments closed under sum-formation, could also be derived by assuming closure under sum-formation occurs in the syntactic derivation itself. In fact, the main claim to fame of the operator approach is that it allows cumulation of *derived* predicates—i.e., of predicates denoted by non-terminal nodes in a syntactic structure—and not just of lexical predicates in the terminal "leaf" nodes of the syntactic tree (cf. Beck & Sauerland 2000). The operator approach is thus more expressive than the lexical one, and so are other alternatives to lexical cumulativity on the market, which were all proposed as ways to derive cumulative inferences that are undergenerated by the lexical approach.[13] These various proposals all agree that we need a mechanism that is *at least* as expressive as the operator approach to account for the range of attested cumulative inferences, with some—notably Schmitt (2019)—faulting the operator approach for not being expressive *enough*.

I will not discuss alternative approaches any further for a simple reason: The goal of this chapter is to illustrate that augmenting our intensional semantics with a cumulation mechanism leads to the overgeneration of an unattested reading; namely, R4. If I can show that overgeneration arises even by adopting the cumulation mechanism whose expressive power is generally agreed in the literature to set a lower bound on what is required to correctly capture the empirical landscape—namely, the operator approach—then this should suffice for the reader to extrapolate that any more expressive mechanism would give rise to the same problem.

[13] These include, for instance, Schmitt's (2019; 2020) theory of *plural projection*, and the account of cumulation in the framework of *event semantics* (Schein 1993; Landman 2000; Kratzer 2003, 2008; Ferreira 2005).

3.5.2 Deriving R4 as a cumulative inference

Now let us be slightly more explicit about the implementation of the intensional systems reviewed in Section 3.4. Recall that we encountered two kinds of frameworks. On the one hand are scope-based theories, like the classical scope theory of intensionality (STI) and its modification in the form of Keshet's (2008) split intensionality, in which both the scope of a DP relative to an intensional operator and the intensional status of its restrictor are determined by the DP's position at LF. On the other hand are world variable systems, in which scope is fully determined structurally, but intensional status is not.

The main dividing line between the two families of approaches has to do with how they determine the evaluation world of a world-dependent lexical item. In variable systems, this is done by binding object language variables that are fed as arguments to such lexical items. In scope-based systems, the evaluation world of a lexical item is determined by its structural position relative to certain operators in the structure. From a technical standpoint, the two approaches differ in what they *minimally* need their type inventory to include; or, more specifically, in whether or not they need a primitive type s of possible worlds as part of their type inventory.[14] Scope-based systems can do without worlds in their type inventory; they need not posit (23d) as a clause in their recursive definition of semantic types, appealing to possible worlds only when constructing modal intensions (23e). Variable-based systems, on the other hand, include both (23d) and (23e), as they use syntactic variables ranging over elements of type s as evaluation indices.

(23) A recursive definition of semantic types:

a. e, for individuals, is a type.

b. t, for truth-values (1 or 0), is a type.

c. If σ and τ are types, then so is $\langle \sigma, \tau \rangle$.

d. s, for possible worlds, is a type.

e. If σ is a type, then so is $\langle s, \sigma \rangle$; i.e., the type of modal intensions (functions from possible worlds of type s to elements of type σ).

f. Nothing else is a type.

Interestingly, despite their differences, there is a particular feature that unites the world variable framework and split intensionality, setting them both apart from STI: While these modifications of STI both allow open-class lexical

[14] See Zimmermann 2018 for a useful discussion.

items or particular nodes at LF that dominate them to denote modal intensions, classical STI does not. I will show that having such intension-denoting nodes at LF is the core ingredient in the derivation of unattested R4s, and then explore whether adopting STI, which does not make these nodes available, can thus block the unattested reading with less stipulations than the other systems require. For expository purposes, I will first show how one can derive R4 in a variable system and in the system of split intensionality, and only then proceed to examine how STI fares in comparison. This is, of course, not how things are usually done in discussions of this sort; e.g., Keshet 2008; von Fintel & Heim 2011. Rather, usually STI and its shortcomings are presented, and modifications are then introduced as offering a way to circumvent these shortcomings. Since I will eventually, tentatively defend STI, it makes sense to do things in the opposite way here; namely, to discuss the system I eventually adopt after demonstrating the shortcomings of its alternatives.

World variables

In a variable system, world-dependent lexical items are functions whose input is a world-argument. So nouns and one-place predicates like "*victim*" or "*trapped*" denote a kind of modal intensions; namely a relation between worlds and individuals (24a-24b). The existential modal "*might*" combines with its own world-argument, and with a contextually supplied accessibility function that maps that world-argument to a set of worlds accessible from it (Kratzer, 1977, 1981), returning a generalized quantifier over worlds (24c); a function true of any proposition if there is an accessible world of which that proposition holds. On the other hand, quantificational determiners, assumed here to include numerals like "*two*" (24d),[15] are world-independent; i.e., they do not take worlds as arguments, nor are their arguments themselves intensional.

(24) a. $[\![\text{victim}]\!] = \lambda w \lambda x.\ x$ is a victim in w (shorthand, $\text{victim}(w, x)$)

b. $[\![\text{trapped}]\!] = \lambda w \lambda x.\ x$ is trapped in w (shorthand, $\text{trapped}(w, x)$)

c. $[\![\text{might}]\!] = \lambda w \lambda f_{\langle s,st\rangle} \lambda p_{\langle s,t\rangle}.\ \exists w' \in f(w) : p(w')$

d. $[\![\text{two}]\!] = \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle}.\ \exists x : |x| = 2 \ \wedge\ P(x) \ \wedge\ Q(x)$
(where $|x|$ refers to the number of atomic elements in x)

In this system, following Heim & Kratzer 1998, sister nodes in a syntactic structure combine via the function application rule in (25a). FA allows a

[15] Nothing in what follows rests on this assumption; the argument to be made below is fully compatible with an analysis of numerals as predicative elements that appear under a silent existential quantifier.

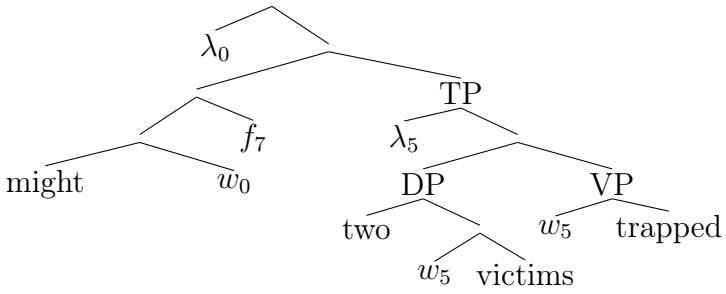


Figure 3.3: LF that derives the *de dicto* reading of (17), with world variables

function-denoting element to combine with its syntactic sister, as long as its sister denotes an element in the domain of the function it denotes. However, given that lexical elements like the noun in (24a) denote modal intensions, whereas determiners like (24d) that take them as complements have only extensional arguments in their domains, something more must be said about how determiners combine with their complements, and more generally, how world-dependent items combine with items that denote functions with extensional domains. Potential type-mismatch is avoided in this case by assuming that world-arguments are always saturated by variables in the object-language, which are effectively, pronouns ranging over possible worlds.

These pronouns do not remain free. Rather, each pronoun is bound by one of multiple λ-binders, positioned immediately below any intensional operator and in a position outscoping the entire matrix sentence, with the aid of two additional composition rules in (25b) and (25c).

(25) a. **Function Application (FA):** If α is a node with daughters $\{\beta, \gamma\}$, then if $[\![\beta]\!]$ is a function whose domain contains $[\![\gamma]\!]$, $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

b. **Pronoun Rule (PR):** If α is a pronoun or variable carrying an index i, then for any assignment function g from indices to denotations, $[\![\alpha]\!]^g = g(i)$

c. **Predicate Abstraction (PA):** If α is a node with daughters $\{\beta, \gamma\}$, and β dominates only an indexed operator λ_i, then $[\![\alpha]\!] = \lambda x.\ [\![\gamma]\!]^{[i \to x]}$

(26) $\lambda w.\ \exists x : |x| = 2\ \wedge\ \text{victims}(w, x)\ \wedge\ \text{trapped}(w, x)$

Figure 3.3 provides the structure that would derive, in this system, the *de dicto* reading of (17), according to which it might be the case that two victims are trapped under the rubble. The world pronouns adjoining to the predicates "*victims*" and "*trapped*" in that structure are co-indexed with, and therefore bound by, the λ-operator under the existential modal "*might*". The

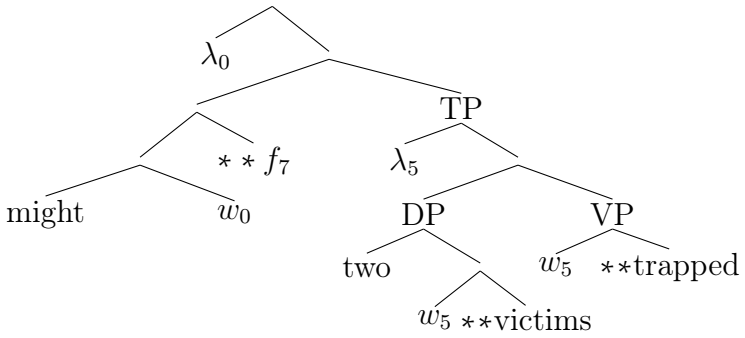


Figure 3.4: LF that derives R4 for (17), with world variable

node labeled TP in the figure thus denotes the proposition in (26), true of a world if there are two trapped victims in that world, and once the modal combines with (26) the entire sentence asserts that there is (at least) one accessible world in which there are two trapped victims.

To derive the unattested R4 of (17)—or rather, a reading of (17) that is strictly weaker than that R4, and thus wrongly predicted to be true in a scenarios like (18)—one only has to insert cumulation operators in the right places in the structure used to derive the *de dicto* reading. This is done in Figure 3.4, where the two-place relations "*victims*" and "*trapped*" as well as the contextually supplied accessibility relation that is an argument of the modal, combine with the pluralizing "double-star" $**$ (21b) before merging with anything else. After combining with $**$, the predicate "*victims*" denotes a relation true of a plurality of worlds w and a plurality of individuals x, just in case for every individual part of x there is a world part of w in which that part is a victim, and for every world part of w there is an individual part of x that is a victim in that world (27a). The same holds, *mutatis mutandis*, for the predicate "*trapped*" (27b).

The contextually supplied accessibility relation, designated as f_7 in Figure 3.4, denotes some function of type $\langle s, st\rangle$ mapping any world to the set of worlds accessible from it. Applying a sum-formation operator to that function will result in a mapping as schematized in (27c); i.e., in a function mapping any world or plurality of worlds w to a set containing all worlds accessible from w and their sums (if w is a single world), or all worlds accessible from parts of w and their sums (if w is a plurality). Note that in the denotation of "*might*" in (24c), the accessibility function is provided with the modal's evaluation world as its first argument, with the resulting set of worlds restricting the existential quantifier introduced by the modal. If we assume that any modal's world argument is always saturated by a single world, *not* a plurality, we can

simplify things by treating the pluralized accessibility function as one that maps any world to the set of worlds accessible from it once that set is itself closed under sum-formation.

(27) a. $[\![**\text{victim}]\!] = \lambda w \lambda x.\ [\forall y \leqslant x : \exists w' \leqslant w : \text{victim}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{victim}(w', y)]$

b. $[\![**\text{trapped}]\!] = \lambda w \lambda x.\ [\forall y \leqslant x : \exists w' \leqslant w : \text{trapped}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{trapped}(w', y)]$

c. For any function f of type $\langle s, st \rangle$, $[\![**]\!](f)$ is the following function: $\lambda w \lambda w'.\ [\forall w'' \leqslant w : \exists w''' \leqslant w' : f(w'', w''')] \wedge [\forall w''' \leqslant w' : \exists w'' \leqslant w : f(w'', w''')]$

The derivation then proceeds as in (28).

(28) a. $[\![\text{DP}]\!]^g = \lambda f.\ \exists x : f(x) \wedge |x| = 2 \wedge [\forall y \leqslant x : \exists w' \leqslant w_5 : \text{victim}(w', y)] \wedge [\forall w' \leqslant w_5 : \exists y \leqslant x : \text{victim}(w', y)]$

b. $[\![\text{VP}]\!]^g = \lambda x.\ [\forall y \leqslant x : \exists w' \leqslant w_5 : \text{trapped}(w', y)] \wedge [\forall w' \leqslant w_5 : \exists y \leqslant x : \text{trapped}(w', y)]$

c. $[\![\text{TP}]\!]^g = \lambda w.\ \exists x : |x| = 2 \wedge [\forall y \leqslant x : \exists w' \leqslant w : \text{victim}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{victim}(w', y)] \wedge [\forall y \leqslant x : \exists w' \leqslant w : \text{trapped}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{trapped}(w', y)]$

d. $[\![\text{might}_{w_0} \ * *f_7]\!]^g = \lambda q.\ \exists w \in [\![**]\!]g(7)(w_0) :\ q(w)$

e. $[\![\text{Figure 3.4}]\!]^g = \lambda w.\ \exists w' \in [\![**]\!]g(7)(w) : \exists x : |x| = 2 \wedge [\forall y \leqslant x : \exists w'' \leqslant w' : \text{victim}(w'', y)] \wedge [\forall w'' \leqslant w' : \exists y \leqslant x : \text{victim}(w'', y)] \wedge [\forall y \leqslant x : \exists w'' \leqslant w' : \text{trapped}(w'', y)] \wedge [\forall w'' \leqslant w' : \exists y \leqslant x : \text{trapped}(w'', y)]$

The TP in Figure 3.4 denotes the proposition in (28c), true of any world or plurality thereof w if there are two individuals each of whom is a trapped victim in some part of w, as long as in every part of w one of these individuals is a trapped victim. Having combined with its world argument and accessibility function (28d), the modal then takes this proposition as its input, asserting it holds of some accessible world or plurality thereof (28e).

Now, consider again the collapse scenario in (18), relative to which (17) is uttered, and assume (17) does, in fact, have the truth conditions we derive for it in (28e). In that scenario, each of two individuals could be victims trapped under the rubble. This means that there is one accessible world in which one individual is a trapped victim, and another accessible world in which another

individual is a trapped victim. Let us call these accessible worlds w and w', and those individuals x and x'. In this scenario, then, the following holds:

(29) a.$< w, x >, < w', x' > \in$ $[\![\text{victims}]\!]$
b.$< w, x >, < w', x' > \in$ $[\![\text{trapped}]\!]$
c.$< w \oplus w', x \oplus x' > \in$ $[\![**\text{victims}]\!]$
d.$< w \oplus w', x \oplus x' > \in$ $[\![**\text{trapped}]\!]$

Since it is stipulated that both w and w' are accessible in the world in which the scenario unfolds, the plurality $w \oplus w'$ is in the set that restricts existential quantification introduced by the modal, given that we use $**$ to close this set under sum-formation. Thus, the truth conditions we derive for (17) in (28e) are verified in the collapse scenario: The plurality of accessible worlds $w \oplus w'$ witnesses the existential quantification over worlds in (28e), and the two-sized plurality of individuals $x \oplus x'$ witnesses the existential quantifier over individual pluralities.

Note that we derive a reading that is strictly weaker than what we define as R4 of (17). On its R4, the sentence is true if two individuals are such that for each there is a possible world in which they are a victim trapped under the rubble. According to the truth conditions in (28e), the sentence is true if there are two individuals and two worlds, such that each individual is a victim in at least one of these worlds, and is trapped in at least one of these worlds, and each of these worlds is such that at least one of the individuals is a victim in it, and at least one individual is trapped in it. This leaves open the possibility that each of the two individuals existentially quantified over is a victim in one world and trapped in another. However, given that the collapse scenario in (18) verifies the strong R4 of (17), it also verifies the weaker reading we derive via cumulation.

Within the world variable framework, then, one can derive a reading for (17) that is predicted to be true in a scenario like (18). Furthermore, this can be done by simply inserting cumulation operators in the structure that derives the *de dicto* reading for the sentence, without actually assigning wide scope for the DP "*two victims*". Thus, the feature of the world variable framework in charge of blocking the derivation of R4 in the first place—namely, the feature preventing elements not in the scope of an intensional operator from receiving opaque evaluation relative to that operator—does not actually do what it is intended to do. Or, rather, while it blocks a particular structure that would give rise to a reading that is true in an exclusively-R4-verifying scenario, it does not block all possible structures that give rise to such a reading.[16]

[16]This suggests that using the term *wide scope opaque* to refer to the relevant unattested

Scope-based systems

I now turn to scope theories of intensionality. As mentioned earlier, I will focus on Keshet's (2008; 2010b) system of split intensionality, but it will be useful to present how that system was developed as a modification of the classical scope system.

The Scope Theory of Intensionality The paradigmatic scopal approach to intensionality is Montague's (1970; 1973) *Scope Theory of Intensionality* (STI), a somewhat modernized textbook version of which is provided in Heim & Kratzer 1998. There, there are no linguistic expressions that denote entities of type s; i.e., no world pronouns. Rather, a world parameter is tacked onto the model's interpretation function in addition to the assignment function parameter g—i.e., $[\![\cdot]\!]^{w,g}$—which maps any item to its *extension* in w. Thus, an NP like "*victims*" is mapped to the set of victims in the world of evaluation (30a). Declarative sentences are mapped to truth values, 1 if true in the evaluation world and 0 otherwise (30b). Finally, intensional operators like modals and attitudes are taken to quantify over interpretative parameters, shifting the world parameter of elements in their scope (30c). The modal in (30c) takes a contextually supplied accessibility function as before, returning a function from propositions to truth values. Note that since the interpretation function is now parameterized, we have to reformulate our composition rules as in (31-33).

(30) a. $[\![\text{victims}]\!]^{w,g} = \lambda x.\ \text{victim}(w, x)$

b. $[\![\text{Two victims are trapped}]\!]^{w,g} = 1$ *iff* $\exists x : |x| = 2 \wedge \text{victim}(w, x) \wedge \text{trapped}(w, x)$

c. $[\![\text{might } f_7]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \exists w' \in g(7)(w) : p(w') = 1$

(31) **Function Application (FA):** If α is a branching node with daughters $\{\beta, \gamma\}$, then for any world w, if $[\![\beta]\!]^w$ is a function whose domain contains $[\![\gamma]\!]^w$, then $[\![\alpha]\!]^w = [\![\beta]\!]^w([\![\gamma]\!]^w)$.

(32) **Pronoun Rule (PR):** If α is a pronoun or variable carrying an index i, then for any world w and assignment function g from indices to denotations, $[\![\alpha]\!]^{w,g} = g(i)$

(33) **Predicate Abstraction (PA):** If α is a branching node with daughters $\{\beta, \gamma\}$, and β dominates only an indexed lambda-operator λ_i, then for any world w, $[\![\alpha]\!]^w = \lambda x.\ [\![\gamma]\!]^{w,[i \to x]}$

reading is misleading: The reading achieves the same effect as one would achieve by assigning wide scope to the numeral "*two*" while evaluating the NP "*victims*" opaquely, but it is shown above that the reading can also be derived without this non-standard scopal configuration.

Now, consider again (17), and suppose we assign to that sentence the LF in (34). In this structure, the modal takes the sentence "*two victims are trapped*" as its syntactic complement. But this gives rise to a type-mismatch: The complement is of type t (30b), whereas the modal is seeking a propositional argument of type $\langle s,t\rangle$ (30c). In such cases, an additional mechanism kicks in, allowing the modal to combine with the modal intension of its complement. This is achieved by positing, in addition to extensional function application (31), a composition rule that introduces modal intensions when so required to avoid a crash of the semantic derivation (35). The derivation of truth-conditions of (34) then proceeds as in (36).

(34) [[might f_7] [$_{\text{TP}}$ [$_{\text{DP}}$ two victims] [$_{\text{VP}}$ trapped]]]

(35) **Intensional Functional Application (IFA):** If α is a branching node with daughters $\{\beta,\gamma\}$, then for any world w, if $[\![\beta]\!]^w$ is a function whose domain contains $[\lambda w'.\ [\![\gamma]\!]^{w'}]$, then $[\![\alpha]\!]^w = [\![\beta]\!]^w(\lambda w'.\ [\![\gamma]\!]^{w'})$.

(36) $[\![(34)]\!]^{w,g}$= $[[\![\text{might } f_7]\!]^{w,g}(\lambda w'.[\![[_{\text{TP}}\ [_{\text{DP}}\ \text{two victims}]\ [_{\text{VP}}\ \text{trapped}]]]\!]^{w',g})]$
$= \exists w' \in g(7)(w) : \exists x : |x| = 2 \wedge \text{victim}(w',x) \wedge \text{trapped}(w',x)$
(substitution from (30b-30c))

IFA—the mechanism that introduces modal intensions into the derivation (35)—is only triggered by the need to provide an intensional operator (i.e., a modal) with the input it is looking for (i.e., a proposition). In other words, modal intensions are only introduced into the derivation when we are interpreting a node immediately dominating an intensional operator. As a result, *all* and *only* elements in the syntactic scope of such an operator will receive opaque evaluation relative to that operator. Therefore, for the LF in (34), in which the DP "*two victims*" is positioned below the modal "*might*", the system only derives truth-conditions that correspond to the *de dicto* reading of (17). To derive a *de re* reading in which the restrictor "*victims*" is evaluated transparently, we need another LF, in which that DP moves to a position above the intensional environment created by the modal, provided in (37). Together with the composition rules that allow us to interpret such movement structures (32-33), and the generalized quantifier semantics for the DP in (38), we end up with the truth-conditions in (39) for (37).[17]

(37) [[$_{\text{DP}}$ two victims] [λ_{15} [[might f_7] [$_{\text{TP}}$ [x_{15}] [$_{\text{VP}}$ trapped]]]]]

(38) $[\![\text{two victims}]\!]^{w,g}$= $\lambda f.\ \exists x : |x| = 2 \ \wedge\ \text{victims}(w,x) \ \wedge\ f(x)$

(39) $[\![(37)]\!]^{w,g} = \exists x : |x| = 2 \wedge \text{victims}(w,x) \wedge \exists w' \in g(7)(w) : \text{trapped}(w',x)$

[17] Derivation omitted for conciseness.

Recall that the main shortcoming of STI, as discussed abstractly in Section 3.4, is its biconditional dependence between quantificational scope and intensional status which undergenerates Fodor's *third reading*, which pairs narrow quantificational scope with transparent evaluation. This should now be immediately observable: Since anything below an intensional operator is interpreted opaquely relative to the operator and anything above it is interpreted transparently, movement of a restrictor is required to achieve transparent evaluation. But, on the common assumption that movement of the restrictor necessitates movement of the determiner that it restricts, in the resulting system a determiner will necessarily take wide scope whenever its restrictor is transparent.

Split Intensionality The undergeneration problem for STI is ameliorated in Keshet 2008, 2010b by a minimal modification which licenses transparent evaluation of elements below an intensional operator, but does not do away with the core principle of STI that syntactic position determines intensional status, and does not enrich the system with world pronouns of type s. Such a system, which Keshet calls *Split Intensionality*, enriches our vocabulary with an operator, ^, defined syncategorematically as in (40), which returns the modal intension of its input. For convenience, I will sometimes say that ^ *intensionalizes* input.

(40) For any expression α and world w, $[\![\hat{}\alpha]\!]^w = \lambda w'.\ [\![\alpha]\!]^{w'}$

This operator replaces IFA as the mechanism responsible for introducing modal intensions, and thus equips us with a way of introducing modal intensions that is not parasitic on the immediate presence of an intension-seeking operator. In other words, it lets us form an intension-denoting node by inserting ^ *anywhere* in the structure below an intensional operator, while still preserving a region in the structure—namely, the region between ^ and the nearest intensional operator above it—where elements are interpreted transparently.

For a less abstract illustration, let us derive in Keshet's system the third reading of, say Fodor's (14), repeated in (41), which contains an attitude predicate as its intensional operator.[18] I assign the LF in (42) to the sentence, in which the DP "*an inexpensive coat*" raises to a position between the attitude "*want*" and ^. This derives, for the sister node of the raised DP, the denotation in (43a), which is a function of type $\langle e, st\rangle$. Given that the DP itself denotes

[18] I must briefly depart here from our example in (17). That sentence involves two existential quantifiers; the modal and the plural indefinite. But these quantifiers are scopally commutative, and so the third reading of the sentence is semantically indistinguishable from its *de re* reading in (39). I therefore take this detour and will return to our target example below.

a generalized quantifier of type $\langle et, t\rangle$, the two cannot combine via FA (31). Rather, an additional composition rule is posited, which lets us derive the proposition in (43b). I will use a simplified version of such a rule, reproduced in (44) from Elliott 2023, which lets us "pass up" the world argument of the function in (43a) before composing it with the generalized quantifier. The proposition in (43b) in turn constitutes the input to the attitude verb, which is thus able to scope above the indefinite DP even though that DP's restrictor is evaluated transparently, resulting in the truth-conditions in (43c).

(41) Mary wants to buy an inexpensive coat.

(42) [Mary [wants [[an inexpensive coat] [7 [^ [PRO_{Mary} to buy x_7]]]]]]

(43) a. $[\![[7\ [\hat{}\ [\text{PRO}_{\text{Mary}}\text{ to buy } x_7]]]]\!]^w = \lambda x \lambda w'.$ Mary buys x in w'

b. $[\![[[\text{an inexpensive coat}]\ [7\ [\hat{}\ [\text{PRO}_{\text{Mary}}\text{ to buy } x_7]]]]]\!]^w =$
$\lambda w'.\ \exists x : x$ is an inexpensive coat in $w \wedge$ Mary buys x in w'

c. $[\![(42)]\!]^w = \forall w' \in \text{DESIRE}_{Mary,w} :$
$\exists x : x$ is an inexpensive coat in $w \wedge$ Mary buys x in w'

(44) **Saturation (SA):** If, for any world w, α is a branching node with daughters $\{\beta, \gamma\}$, with $[\![\beta]\!]^w$ a function of type $\langle et, t\rangle$ and $[\![\gamma]\!]^w$ a function of type $\langle e, st\rangle$, $[\![\alpha]\!]^w = \lambda w'.\ [\![\beta]\!]^w(\lambda x.\ [\![\gamma]\!]^w(x)(w'))$.

Now let us return to our target sentence in (17). Its *de dicto* reading, within Keshet's system, is derived almost exactly in the same way as it is derived in STI, illustrated in (34-36). The only difference, of course, is that in the modified system there is no IFA rule, so modal intensions are introduced by inserting the ^ operator at LF, as in (45).

(45) [[might f_7] [^ [$_{\text{TP}}$ [$_{\text{DP}}$ two victims] [$_{\text{VP}}$ trapped]]]]

As for R4, there are several technically interesting ways to derive this unattested reading for (17) in Keshet's system, of which I present next the one that would require what I think is the simplest discussion. All of the possible derivations require a slightly more significant departure from the *de dicto* LF in (45), relative to the what we have seen above in our discussion of the world variable framework. What is important, though, is that the reading is still derivable from a structure in which the DP "*two victims*" does not outscope the existential modal, by inserting cumulation and ^ operators in the right positions, and slightly expanding the set of composition rules that let us "pass up" world arguments, which Keshet uses as well.

One LF that would derive R4 for (17) in this system is provided in Figure 3.5. In this structure, we use the intensionalizing operator ^ to create nodes

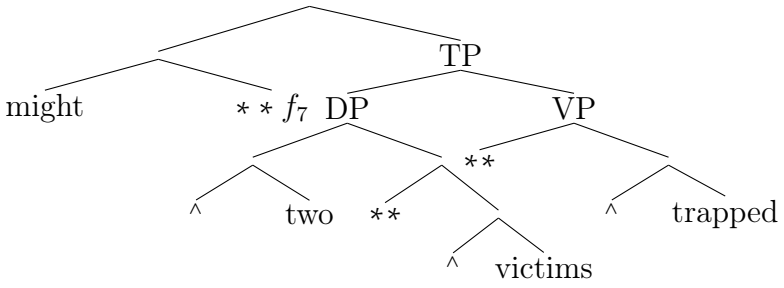


Figure 3.5: LF that derives R4 for (17), in Keshet's scope system

that denote modal intensions, which are then supplied as input to a cumulation operator. More specifically, ^ attaches to the lexical predicates "*victims*" and "*trapped*", which in any scope system denote functions of type $\langle e,t\rangle$ (30a), resulting in modal intensions of type $\langle s,et\rangle$, as spelled out in (46).

(46) a. $[\![\text{\^{}victims}]\!]^w = \lambda w'\lambda x.\ \text{victims}(w',x)$
b. $[\![\text{\^{}trapped}]\!]^w = \lambda w'\lambda x.\ \text{trapped}(w',x)$

Of course, attaching ^ to a lexical item in the split intensionality system results in the same kind of modal intension that constitutes the denotation of that same lexical item in the world variable system; e.g., $[\![\text{\^{}victims}]\!]^w$ in (46a) is just what we have in (24a) for $[\![\text{victims}]\!]$. And we have already seen, in our discussion of the world variable system, the results of applying ∗∗ to such modals intensions. These are repeated in (47a-47b) below, now as the denotation of "double-starred" intensionalized predicates.

(47) a. $[\![\ast\ast\text{\^{}victim}]\!]^w = \lambda w'\lambda x.\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{victim}(w'',y)] \wedge [\forall w'' \leqslant w' : \exists y \leqslant x : \text{victim}(w'',y)]$ (≈ (27a))
b. $[\![\ast\ast\text{\^{}trapped}]\!]^w = \lambda w'\lambda x.\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{trapped}(w'',y)] \wedge [\forall w'' \leqslant w' : \exists y \leqslant x : \text{trapped}(w'',y)]$ (≈ (27b))

With these denotations for the pluralized intensionalized predicates, there are two potential type-mismatches in Figure 3.5. We run into the first when attempting to combine the modal intension of type $\langle s,et\rangle$ in (47a), within the DP in Figure 3.5, with the determiner "*two*". As before, we still treat the determiner as just a generalized quantifier of type $\langle et,\langle et,t\rangle\rangle$ (48). However, we also attach to it an intensionalizing operator, which vacuously binds a world argument (49), turning it into a function of type $\langle s,\langle et,\langle et,t\rangle\rangle\rangle$. This step does not, by itself, allow us to avoid type-mismatch as modal intensions of type $\langle s,et\rangle$ are not in the domain of functions of type $\langle s,\langle et,\langle et,t\rangle\rangle\rangle$. However, it

does allow us to posit a composition principle, which effectively lets us "ignore" the world argument, by passing it up to be saturated later in the derivation. This is spelled out in (50),[19] which lets us derive (51) as the denotation of the DP in Figure 3.5.

(48) $[\![\text{two}]\!]^w = \lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}.\ \exists x : |x| = 2\ \wedge\ P(x)\ \wedge\ Q(x)$

(49) $[\![\text{\^{}two}]\!]^w = \lambda w'.\ [\![\text{two}]\!]^{w'} = \lambda w'\lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}.\ \exists x : |x| = 2\ \wedge\ P(x)\ \wedge\ Q(x)$

(50) **World Percolation (WP):** If, for any world w and any types σ, τ, α is a branching node with daughters $\{\beta, \gamma\}$, with $[\![\beta]\!]^w$ a function of type $\langle s, \langle \sigma, \tau\rangle\rangle$ and $[\![\gamma]\!]^w$ a function of type $\langle s, \sigma\rangle$, $[\![\alpha]\!]^w = \lambda w'.\ [\![\beta]\!]^w(w')([\![\gamma]\!]^w(w'))$.

(51) $[\![\text{DP}]\!]^w = \lambda w'\lambda P_{\langle e,t\rangle}.\ \exists x : |x| = 2\ \wedge\ P(x)\ \wedge [\forall y \leqslant x : \exists w'' \leqslant w' : \text{victim}(w'', y)]\ \wedge\ [\forall w'' \leqslant w' : \exists y \leqslant x : \text{victim}(w'', y)]$

We can now appeal to WP to avoid a second potential type-mismatch, when attempting to combine the DP in (51), of type $\langle s, \langle et, t\rangle\rangle$, with the VP, which is again a modal intension of type $\langle s, et\rangle$. WP lets us derive the denotation in (52) for the TP in Figure 3.5, again percolating up the world argument, and run-of-the-mill function application then derives (53) as the result of combining that TP with the existential modal.

(52) $[\![\text{TP}]\!]^{w,g} = \lambda w'.\ \exists x : |x| = 2\ \wedge\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{victim}(w'', y)]\ \wedge\ [\forall w'' \leqslant w' : \exists y \leqslant x : \text{victim}(w'', y)]\ \wedge\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{trapped}(w'', y)]\ \wedge\ [\forall w'' \leqslant w' : \exists y \leqslant x : \text{trapped}(w'', y)]$

(53) $[\![\text{Figure 3.5}]\!]^{w,g} = [\![\text{might } f_7]\!]^{w,g}([\![(52)]\!]^{w,g}) = 1$ *iff*
$\exists w' \in [\![**]\!]g(7)(w)$: $\exists x$: $|x| = 2\ \wedge\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{victim}(w'', y)]\ \wedge\ [\forall w'' \leqslant w' : \exists y \leqslant x : \text{victim}(w'', y)]\ \wedge\ [\forall y \leqslant x : \exists w'' \leqslant w' : \text{trapped}(w'', y)]\ \wedge\ [\forall w'' \leqslant w' : \exists y \leqslant x : \text{trapped}(w'', y)]$

As already illustrated in our discussion of the world variable system above, the truth conditions in (53) are met in the collapse scenario (18), which verifies exclusively the R4 of (17). Therefore, if the structure in Figure 3.5 is available as a LF for (17), and the additional composition principle we posit

[19] Note that while I define WP in (50) in a way that is specifically tailored to our needs in this section, it is actually an instantiation of a general operation in combinatory logic (the S-combinator; cf. Seldin 2006), that has been appealed to in work within combinatorial grammar (cf. Steedman 1987, 1988; Szabolcsi 1989). Note further that the same instantiation is appealed to in, e.g., Schmitt's (2020) work on the semantics of attitude constructions.

here—namely, WP (50)—is legitimate, then the split intensionality system overgenerates unattested instances of R4, just like the world variable system.[20]

3.5.3 Taking stock

We have seen how two different approaches to the representation of intensionality overgenerate the unattested R4 once augmented with a mechanism that derives cumulative inferences. While the derivations of R4 differ in their technicalities under each approach, they share a simple common thread. In both cases, what enabled the generation of R4 is the presence of nodes at LF which denote two kinds of modal intensions:

(i) relations between worlds and individuals contributed by nouns or verbs like "victim" or "trapped", and

(ii) functions from worlds to truth values, serving as restrictors of modals like "might".

Closure of these modal intensions under sum-formation let us derive R4 as a cumulative reading. We have assumed, as is standard, that in both systems intensions of the type described in (ii) are simply supplied contextually by

[20] A potential response on behalf of Split Intensionality might target our distribution of intensionalizing operators in Figure 3.5. For instance, ^ is applied vacuously to the determiner "*two*", and one could bar this kind of move. Alternatively, one may argue that the ^ operators we attach to predicates of type $\langle e, t\rangle$ are illicit. These operators create two nodes that denote relations of type $\langle s, et\rangle$, which are then pluralized to achieve R4. However, all that Keshet really needs to derive Fodor's (1970) third reading, is to place ^ in a position whose sister is a node of type t as in (42), creating a proposition-denoting node. And while no deep principle in the system restricts ^ to such positions, perhaps our derivation can be blocked by positing such a restriction. There may even be independent motivation for doing so: Larson (2002) and den Dikken, Larson, & Ludlow (2018) argue, from the behavior of so-called intensional transitive verbs, that intensionality is grammatically conditioned in that it is introduced only at the level of clausal complements, suggesting that only syntactic clauses may be intensionalized (I thank Kai von Fintel for pointing this out to me). I will not engage with this point here beside noting that it is quite controversial (for instance, Forbes (2000, 2006); Zimmermann (2006) present well-known alternative views; see also Schwarz 2021 for an overview of the debate and further references). While further research, beyond the scope of this chapter, is required to explore whether this is a viable response to the challenge of R4, note that even if it is, such a response requires one be committed to a scope system as the correct framework with which to account for the world-dependence of utterances. This, in itself, is a notable result, and is particularly relevant for the argument to be made in the second part of the chapter, which defends the position that we need to allow cumulation to interact with tense and temporal operators: Even if we manage to block R4 under split intensionality, our conclusion that we need to treat worlds and times differently in our intensional system to deal with tense, will still stand.

syntactic variables that range over functions of type $\langle s, t \rangle$. As for the intensions described in (i), these are by definition the denotations of nouns and intransitive verbs items under the world variable approach, and are derived via an intensionalizing operator at LF under split intensionality.

Our findings provide new support for Schmitt's (2023) conclusion that there is a ban on cumulative readings with worlds, as such a ban would be needed to block the derivation of R4, under either theory of intensionality. Observe that, as noted earlier, unlike Schmitt's original argument from the lack of cumulative readings of conditionals and other particular modal constructions, the argument above cannot be bypassed by stipulating that particular lexical items behave in a particular way. Rather, a more general stipulation would be needed, prohibiting (at the very least) any modal intension of the kind in either (i) or (ii) from undergoing closure under sum-formation. An immediate question that arises is, of course, what accounts for this ban?

Next, I explore whether adopting STI as our representation of intensionality can help us derive the ban from independent principles. This is a promising idea, given that the main difference between STI and the two theories we have considered, which are themselves quite different from each other, is that under STI no LFs nodes denote the modal intensions of the type described in (i). Rather, lexical items like "victim" or "trapped" denote extensions, and are intensionalized only in the semantics by the IFA composition rule that effectively functions as a "repair strategy", kicking in when required to avoid type-mismatch. Given that closing the modal intensions of the kind described in (i) under sum-formation is a necessary component in our derivations above, we should naturally wonder whether R4 may be blocked simply in virtue of the architecture of STI.

The conclusion I will arrive at is that, alas, STI does not block R4 *just* in virtue of its architecture. However, there may yet be a way of blocking the unattested reading under STI by appealing to more general principles than those that must be appealed to in order to block the reading in the other frameworks we have considered. I will end this part of the chapter by mentioning some issues that this approach raises.

3.6 Attempting to block R4 within the classical Scope Theory of Intensionality

What would be required to close a relation between worlds and individuals under sum-formation in a framework that does not make available LF nodes denoting such relations? I.e., what are the operations that one would need

to block to avoid the generation of R4 under STI? There seem to be two possibilities. First, if the operators of the kind we have been using to derive cumulative inferences (i.e., $*$, $**$ etc.) can themselves have an intensional type, they would trigger the application of the intensionalizing composition rule IFA when combining with their argument, resulting in the sum-closure of the *intension* of their argument. Alternatively, suppose we assume the view termed *lexical cumulativity* in the literature, according to which predicates are "born" as closed under sum-formation. Lexical cumulativity itself might allow cumulation not only over subcategorized syntactic arguments, but over the world parameter as well. Let us review these two possibilities.

3.6.1 Intensional cumulation operators

Suppose we posit a cumulation operator that is itself a function seeking an intensional input; e.g., $**_{\text{INT}}$ in (54a). Merging this operator with the predicate "*victims*" in (54b) would trigger IFA (35) and derive the same function we derive with $[\![**\text{^victims}]\!]^w$ in (47a) within the system of split intensionality, or with $[\![**\text{victims}]\!]$ in (27a) within the world variable framework.

(54) a. $[\![**_{\text{INT}}]\!]^w = \lambda\mathcal{R}_{\langle s,\langle e,t\rangle\rangle}\lambda w'\lambda x.\ \exists\mathcal{R}'_{\langle s,\langle e,t\rangle\rangle} : [\forall w'', x' : \mathcal{R}'(w'', x') \rightarrow \mathcal{R}(w'', x')] \wedge w' = \bigoplus\{w \mid \exists x : \mathcal{R}'(w, x)\} \ \wedge \ x = \bigoplus\{x \mid \exists w : \mathcal{R}'(w, x)\}$

b. $[\![\text{victims}]\!]^{w,g} = \lambda x.\ \text{victim}(w, x)$

c. $[\![**_{\text{INT}}\ \text{victims}]\!]^w =_{\text{(by IFA)}} [\![**_{\text{INT}}]\!]^w(\lambda w'.\ [\![\text{victims}]\!]^{w'})$

Now, some non-trivial technical maneuvers—of the kind we had to resort to in deriving R4 in the framework of split intensionality—would be required to combine the resulting predicate, of type $\langle s, \langle e, t\rangle\rangle$ any further with a determiner like "*two*". But even if we find a plausible way to avoid type-mismatch, there is still a natural way of thinking about cumulation operators that would prevent us from deriving (54c) to begin with, but only under STI.

Cumulation operators, like $*$ or $**$, must be polymorphic; i.e., must be able to apply to elements of different types (cf. Krifka 1986). However, how should we model their polymorphic nature? There are two possibilities. On the one hand, we may simply assume that there are multiple entries for $*$, $**$ etc. in the lexicon, such that for any conjoinable type σ (i.e., any type that ends with t), there is a $*$ or $**$ of type $\langle\sigma, \sigma\rangle$. This would mean that we have $**$ of, say, type $\langle\langle e, et\rangle, \langle e, et\rangle\rangle$, but also of type $\langle\langle s, et\rangle, \langle s, et\rangle\rangle$ as in (54a). On the other hand, we may assume that for an operator, being polymorphic means not having the lexicon fully determine its type; rather a polymorphic operator's type is only determined by the elements with which it combines in

the syntax. This view is implicit in our entries for $*$ and $**$ in (21), repeated below, where instead of type labels, we use variables σ, τ over types.

(55) a. $[\![*]\!] = \lambda\mathcal{F}_{\langle\sigma,t\rangle}\lambda x_\sigma.\ \exists\mathcal{F}'_{\langle\sigma,t\rangle} : [\forall x' : \mathcal{F}'(x') \rightarrow \mathcal{F}(x')] \ \wedge\ x = \bigoplus\{x \mid \mathcal{F}'(x)\}$

b. $[\![**]\!] = \lambda\mathcal{R}_{\langle\sigma,\langle\tau,t\rangle\rangle}\lambda x_\sigma\lambda y_\tau.\ \exists\mathcal{R}'_{\langle\sigma,\langle\tau,t\rangle\rangle} : [\forall x', y' : \mathcal{R}'(x', y') \rightarrow \mathcal{R}(x', y')] \wedge$
$x = \bigoplus\{x \mid \exists y : \mathcal{R}'(x, y)\} \ \wedge\ y = \bigoplus\{y \mid \exists x : \mathcal{R}'(x, y)\}$

On this view, there can be an operator $**$ of type $\langle\langle s, et\rangle, \langle s, et\rangle\rangle$ like the one in (54a) only if there are nodes whose denotations are of type $\langle s, et\rangle$ in the syntax. And since STI does not let us construct such nodes to begin with, pairing this view of the polymorphic nature of cumulation operators with STI does not generate unattested cases of R4.[21] Notably, unlike the stipulation that specifically prohibits modal intensions of type $\langle s, et\rangle$ from being closed under sum-formation, which would be required to block R4 as a cumulative inference under the world variable or split intensionality framework, under STI we can posit a general and natural constraint on polymorphic operations, from which the more specific stipulation immediately follows. Let us now turn to the more challenging case.

3.6.2 Lexical cumulation over the world parameter

We have chosen to derive cumulative inferences with operators like $*$ and $**$ because almost everyone in the literature seems to agree that we need a cumulation mechanism with, *at least*, the expressive power of the operator system to correctly capture the empirical landscape. The logic behind this choice, spelled out in Section 3.5.1, is that given the consensus that operators set the lower bound on required expressive power, if we can show that operators run into an overgeneration problem, that problem will apply to any more expressive system as well. We also mentioned another view, that of *lexical*

[21] The idea that the type of polymorphic operators is only determined once their argument position is saturated can be implemented by revisiting the idea that such elements are *operators* in the syntax to begin with. Instead, we can introduce the semantics of so-called "polymorphic operators" as composition rules. For instance, $**$ can be introduced with a rule defined as follows:

(i) **$**$-Rule:** If, for any world w and any types σ, τ, α is a non-branching node with daughter β, such that $[\![\beta]\!]^w$ is a function of type $\langle\sigma, \langle\tau, t\rangle\rangle$, one of the following holds:

- $[\![\alpha]\!]^w = [\![\beta]\!]^w$; or
- $[\![\alpha]\!]^w = \lambda x_\sigma\lambda y_\tau.\exists\mathcal{R}'_{\langle\sigma,\langle\tau,t\rangle\rangle} : [\forall x', y' : \mathcal{R}'(x', y') \rightarrow [\![\beta]\!]^w(x', y')] \wedge$
$x = \bigoplus\{x \mid \exists y : \mathcal{R}'(x, y)\} \ \wedge\ y = \bigoplus\{y \mid \exists x : \mathcal{R}'(x, y)\}$

I thank Viola Schmitt and Jad Wehbe for discussing this point with me.

cumulativity, according to which predicates are closed under sum-formation as part of their lexical meaning. That view, we noted, is subsumed by the operator approach, because any cumulative inference derived by assuming that the arguments of some predicate are closed under sum-formation in the lexicon, could also be derived by assuming closure under sum-formation occurs via an operator adjoining that predicate at LF.

However, once we introduce new assumptions that effectively constrain the expressive power of the operator system, our assumption that this system is strictly more expressive than the lexical view should be reexamined. Indeed, as Viola Schmitt noted to me in personal communication, complementing the operator approach with the lexical view might allow the former to make up for whatever expressive power it loses by pairing STI with the view of polymorphic operators described above. This is because, if closure under sum-formation is a lexical property of predicates, nothing in principle blocks this closure from targeting the world parameter of a predicate in addition to its subcategorized arguments; i.e., the lexical entries for items like "victim" and "trapped" may be defined along the lines of (56a-56b), and similarly the $\langle s,t\rangle$ restrictor of a modal like "might" might itself be "born" as closed under sum-formation. This will suffice to derive R4 truth conditions for our target sentence in (17).[22]

(56) a. $[\![\text{victim}]\!]^w = \lambda x.\ [\forall y \leqslant x : \exists w' \leqslant w : \text{victim}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{victim}(w', y)]$

b. $[\![\text{trapped}]\!]^w = \lambda x.\ [\forall y \leqslant x : \exists w' \leqslant w : \text{trapped}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{trapped}(w', y)]$

At first blush, STI itself may need a stipulation—say, that the world parameter is never cumulated over—that appears just as *ad hoc* as the one required under split intensionality or the world variable framework.[23] However, there is

[22] Here is how: Assuming the structure from which we derived the *de dicto* reading of (17) under STI, repeated from (34) in (i-a), and the entries in (56a-56b) for "victims" and "trapped", we derive (ii-b) as the denotation of TP. This is a truth value which combines with "might" via IFA. As long as the restrictor of "might", contributed by the free pronoun f_7 in (i-a) contains all epistemically-accessible worlds and their pluralities, the sentence thus asserts that there is a (plurality of) worlds of which the proposition $\lambda w.[\![\text{TP}]\!]^{w,g}$ holds, and this in turn is the case in the R4-scenario that falsifies (17).

(i) a. [[might f_7] [$_{\text{TP}}$ [$_{\text{DP}}$ two victims] [$_{\text{VP}}$ trapped]]]

b. $[\![\text{TP}]\!]^{w,g} = 1$ iff $\exists x : |x| = 2 \wedge [\forall y \leqslant x : \exists w' \leqslant w : \text{victim}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{victim}(w', y)] \wedge [\forall y \leqslant x : \exists w' \leqslant w : \text{trapped}(w', y)] \wedge [\forall w' \leqslant w : \exists y \leqslant x : \text{trapped}(w', y)]$

[23] Of course, a constraint blocking lexical cumulation over a world parameter or argument

suggestive evidence that, here too, general principles could help rule out lexical cumulation also over the world parameter. This is because there may be reasons to reject lexical cumulativity altogether. In particular, Wehbe (2023, 2026), building on Ionin & Matushansky (2002), recently argued that cumulation should be derived only via syntactic operations, never in the lexicon, and conjectures that this is due to a general constraint on the shape of the lexicon, preventing certain logical operations—like closure under sum-formation—from ever being carried out in the lexicon, and thus preventing certain meanings from ever being lexically encoded.

The argument is quite intricate, so I will only reproduce here its core observation, involving the Lebanese Arabic *double subject construction*, exemplified in (57a), in which one thematic subject (Rasha and Hadi), is realized with two DPs. This construction is argued to differ structurally from run-of-the-mill sentences with a plural subject like (57b) only in the position of the conjoined DP subject; in (57a), that DP is in a position below the surface position of the verb, and non-standard movement of a single conjunct above the verb results in the observed word order, whereas in (57b) the conjoined DP itself moves to a pre-verbal position. It is then observed that the structural difference is correlated with a difference in meaning: While (57b) can be true if Rasha and Hadi danced either together (one with the other), or separately, (57a)—with the double subject—is only verified when the dancing was collective.

(57) a. **Rasha** ra?asit **hiyye w Hadi.**
Rasha danced.3SG.F **her and Hadi**
'Rasha and Hadi danced with each other.'
Simplified structure: [Rasha [$_{XP}$ λ_1 [danced [her$_1$ and Hadi]]]]

b. **Rasha w Hadi** ra?aso.
Rasha and Hadi danced.3PL
'Rasha and Hadi danced.'
Simplified structure: [[Rasha and Hadi] [$_{XP}$ λ_1 [danced t_1]]]

But if it is a lexical property of the verb 'dance' that whenever it holds of Rasha and Hadi separately, it holds of their conjunction, the difference in meaning is not predicted: In that case, the node labeled XP in (57a) would denote a predicate true of any individual x if the sum $x \oplus$Hadi is in the denotation of the verb 'dance'. This predicate holds of Rasha as long as she and Hadi danced, separately *or* together, failing to derive the unique meaning of (57a).

is required under *any* of the intensional systems we discuss: If lexical cumulativity lets us close under sum-formation modal intensions in the world-variable system, then regardless of how we manage to block cumulation operators from applying to these intensions, we will still derive R4. The same point applies, *mutatis mutandis*, to Keshet's (2008) system.

Wehbe argues that these facts support the following claims. First, that sum-formation never takes place in the lexicon; rather, a plurality is in the denotation of a lexical predicate only if the predicate holds of the members of this plurality collectively (in the case of 'dance', only if the members of the plurality danced together). Second, that sum-formation is syntactically restricted to derived predicates; thus, in (57a)-(57b) only the nodes labeled XP can be the input of cumulation operators like $*$ and $**$, but not the lexical predicate 'dance' itself.[24] Given these assumptions, the node labeled XP in (57a) denotes a predicate true of an individual x if x and Hadi danced collectively. In (57b), XP denotes a predicate true of an individual or plurality x if x danced, and since XP itself can be closed under sum-formation via $*$ it will hold of the plurality of Rasha and Hadi as long as both individuals danced, separately or together. Wehbe conjectures that the absence of lexical cumulativity may be due to an economy constraint on the lexicon, ruling out lexicalizing meanings that can be derived by the application of an independently needed operator. And given that a sum-closure operator is independently needed (Beck & Sauerland, 2000), it must be solely responsible for closure under sum-formation.

I review this argument here because if its conclusions are warranted, then

[24]One may legitimately protest here, wondering whether Wehbe's assumption that cumulation operators only apply to derived predicates does not by itself already rule out the derivation of the R4 of (17) under the world-variable system, which would render moot our entire discussion of STI. To derive R4 with world variables, we posited the structure in Figure 3.4, where $**$ applies directly to lexical predicates denoting modal intensions like "*victims*" and "*trapped*". However, we can introduce certain technically creative abstraction steps to the structure in Figure 3.4, which will effectively form derived predicates equivalent in meaning to the lexical predicates that, according to Wehbe, cannot be closed under sum-formation. For instance, suppose that instead of mapping the DP "*two victims*" to the structure in (i-a) as we do in Figure 3.4, we assign to it the structure in (i-b), where the determiner "*two*" had moved a short distance, leaving behind an individual-type trace, and the world variable in the restrictor then "tucked-in" between the determiner and its abstractor to create another abstraction node, over worlds. The resulting derived predicate is of type $\langle s, et\rangle$ and can be closed under sum-formation via $**$, as in (i-b).

(i) a. λ_5 … [$_{\text{DP}}$ two [$**$victims w_5]]

b. λ_5 … [$_{\text{DP}}$ two [w_5 [$* * \lambda_4$ λ_7 [t_7 [victims t_4]]]]]

Wehbe argues at length that this kind of technical maneuver is unavailable for the Lebanese double-subject construction, by appealing to generalized economy considerations which in turn do not apply in our case. What is important to note here is that, as previewed, I will argue below that we need to be able to generate R4 relative to tense operators. Thus, if Wehbe is correct that directly applying $**$ to a lexical predicate is ruled out, a technical maneuver of the kind discussed above must be available in the temporal cases.

		The *third* reading	R4
What is attested		✓	✗
What is predicted by...	STI (+assumptions)	✗	✗
	split intensionality	✓	✓
	world variables	✓	✓

Table 3.2: Attested *vs* predicted for each theory of intensionality, together with our additional assumptions regarding cumulativity

while the principles of STI do not by themselves block the derivation of R4, they may be able to do so together with general constraints on cumulation. Note, however, that the logic implied by Wehbe's economy constraint, ruling out closure under sum-formation in the lexicon as the same meanings can be derived via the application of an operator, does not translate so neatly to our discussion here: If we adopt STI, then the meaning that is derived by cumulating over the world parameter in the lexicon is one that, as we have seen, cannot be derived by the application of a syntactic operator (on some natural assumptions about the nature of the operator involved). So while we must accept the conclusion that lexical cumulativity must be ruled out, we must reevaluate the reason that Wehbe conjectures is behind this fact. The consequences of accepting Wehbe's argument to motivate the adoption of STI must therefore be more carefully examined; an endeavor that I leave for future research. In the last part of this section, I will explore some prospects and worries of pursuing this idea.

3.6.3 Prospects and worries

Even if the absence of R4 turns out to be the result of STI and the general constraints on cumulativity discussed above, STI will still face a serious empirical challenge; of the three attested readings of DPs in intensional contexts—i.e., *de re*, *de dicto*, and Fodor's so-called *third* reading—it generates only the first two. The third reading is ruled out by the biconditional dependence between scope and intensional status, ruling out transparent evaluation of restrictors of narrow scope DPs. Thus, we are faced with the picture represented in Table 3.2 below, which shows that each theory of intensionality either overgenerates R4, or undergenerates the third reading.

Interestingly, the idea that we should adopt STI to explain why cumulation mechanisms do not lead to the overgeneration of R4, fits well with a new trend in the literature on intensionality, which has converged on the position that

there are independent reasons to posit an additional interpretative mechanism for intensional sentences; one that would derive the third reading and can supplement STI to avoid undergeneration. Kaufmann (in Schwager 2009), for instance, argues that not all instances of the third reading reduce to shifting the world index of a DP in the scope of an intensional operator, contra the way we have been talking about that reading so far, and the way Keshet (2008) and Percus (2000) talk about it when proposing their alternatives to STI.[25] This means that *all* of the theories observed so far undergenerate at least some instances of the third reading.

To see this, consider the utterance in (58b), which seems true in the context of (58a). Its *de re* reading is false in that context, since there is no specific building Mary wants to buy. Thus, the DP "*a building with 192 floors*" must scope below the attitude verb "*want*". Its *de dicto* reading is also false, since Mary (by assumption) does not know how many floors the Burj Dubai has. Thus, the NP "*building with 192 floors*" must not be evaluated opaquely relative to "*want*". This leaves us with the *third* reading. But if that reading simply pairs narrow scope for a DP with opaque evaluation of its restrictor, as formalized in (58c), (58b) should attribute to Mary inconsistent desires: the third reading truth conditions in (58c) hold only when Mary desires the contradiction $\exists x \in \varnothing$ to hold, as there is no building with 192 floors in the evaluation world.

(58) a. Context: Mary is looking at the Burj Dubai. Not knowing that it has 191 floors and is the world's highest building, she thinks, 'I want to buy a building that's a floor higher!'

b. Mary wants to buy a building with 192 floors.

c. $[\![(58b)]\!]^w = 1$ *iff* $\forall w' \in \text{DESIRE}_{Mary,w}[\exists x\text{: } \textit{has-192-floors}_w(x) \wedge \textit{buy}_{w'}(Mary, x)]$

d. Mary wants to buy a building that is one floor higher than the Burj Dubai.

The fact of the matter is, however, that (58b) does not seem to convey that Mary has inconsistent desires. Rather, it seems to convey something along the lines of what (the *de dicto* reading of) (58d) does, which cannot be captured under the approach to the third reading taken for granted so far.

Kaufmann derives the correct reading of (58b), i.e., the *de dicto* reading of (58d), by positing a mechanism for property substitution that effectively allows us to replace "*building with 192 floors*" in (58b) with "*building that is one floor higher than the Burj Dubai*", and to evaluate the sentence *de dicto*

[25] In fact, Kaufmann's position is that *none* of the instances of the third reading involve narrow scope and transparent evaluation.

after replacement has taken place. A flurry of recent work has attempted to characterize the substitution mechanism required for Kaufmann's proposal to work.[26] What matters for our purposes is just that, whatever the mechanism is, it is able to supplement STI in a way that takes the edge off the undergeneration challenge presumably posited by the third reading. And there seems to be a consensus that, in fact, it is able to do so; i.e., that once we have in place a substitution mechanism, we can—and perhaps should—use it beyond (58b) to derive all instances of the third reading. Now, since no theory we have examined so far is able to account for (58b), they all need to posit this additional mechanism, and given our suggestion that STI can be restricted in a general way so as to not overgenerate R4, the alternative theories may turn out to be inferior to STI.

Many challenges remain. The most important one has to do with the fact that the step of adopting STI itself may be a stipulation. Simply calling something a parameter, rather than a variable, does not *explain* what the nature of this object is and how it differs from a variable. Therefore, even if we manage to derive the absence of R4 from STI in conjunction with general constraints on cumulation, the adoption of STI itself is a stipulation that is unique to worlds, unless we have something explanatory to say about the distinction between variables and parameters. Until a deeper explanation of the distinction between parameters and variables is provided, using STI to rule out R4 remains, in a sense, a technical exercise.[27]

3.7 R4 relative to tense

3.7.1 Some preliminaries

Next, I argue that unlike R4 relative to modals, which is not attested, R4 relative to tense operators *is* attested, and should be derived as a cumulative reading. In our discussion of the modal cases, we have been using the label R4 rather liberally, saying that a sentence has this reading if it is verified by scenarios that do not verify its *de dicto*, *de re*, or *third* readings, but *do* verify the reading that would be derived by assuming that some DP scopes above a modal in the sentence despite being opaque relative to the modal. Above, we have shown that while this reading would be derived if the unavailable wide-scope opaque construal were available, it is *also* derived by applying cu-

[26] See, in particular, Percus 2020; Blumberg & Lederman 2021; Benbaji-Elhadad 2023; Mayr & Schmitt 2024, 2026b

[27] I thank Viola Schmitt for discussing this point.

mulation operators to modal intensions, an operation that needs to be blocked given that the reading does not seem to be attested. On this use of the term R4, it is actually quite easy to show that R4 relative to tense is attested, and this is hardly novel. Consider, say, the following example adapted from Szabó 2010.

(59) a.**Scenario:** While touring the grounds of a seventeenth century courthouse, no longer in use, the tour guide says:

b.Fifteen judges presided in this court.

This sentence seems true in its respective scenario. However, it is not true in virtue of its *de dicto* reading, in which the DP "fifteen judges" scopes below sentential tense: It is certainly not the case that there is some past time point during which fifteen *then* judges presided in court. The sentence is also not true *de re*: Since all judges are no longer among us, there are no fifteen *current* judges, such that for each there is a past time during which they presided in court.[28] Rather, the sentence seems to admit a reading according to which for each of fifteen individuals, there is a past time point during which they were presiding judges. So, there: R4 relative to tense is apparently attested.

What requires more work is demonstrating that R4 of temporal sentences must be derived via those cumulation mechanisms that should be prevented from applying in the modal case. This is because there is an alternative mechanism, which has nothing to do with cumulation, that can derive the truth of a sentence like (59b) in a scenario like (59a), suggesting that the case in (59) does not bear on the question we are interested in. In particular, it is known since Enç (1982, 1986) that at least some NPs can be *temporally-independent* of their host clause; i.e., that they need not always be evaluated relative to clausal tense. For instance, "*fugitive*" in (60) need not be evaluated contemporaneously with the predicate "*is in prison*", as evidenced by (60) being non-contradictory. It can convey that every relevant individual who *at some point* was a fugitive is in prison at the time of utterance, rather than the nonsensical claim that every fugitive at the time of utterance is also in prison at the time of utterance.

(60) Every fugitive is in prison.

Enç's observation suggests that a noun like "*fugitive*" can denote, at least in certain environments, the set of *one-time* fugitives.

If the noun "*judges*" in (59b) behaves like Enç's "*fugitive*", the sentence can convey that for each of fifteen *one-time* judges there is a past interval during

[28]If we assume that both past tense and the numeric determiner "fifteen" are existential quantifiers, the *third* reading is equivalent to the *de re* reading, so we can set it aside.

which they presided in court. This, in turn, is asymmetrically entailed by there being fifteen individuals, such that for each there is a past time during which they were presiding judges (i.e., the paraphrase of the unavailable wide-scope opaque reading): If for each of fifteen individuals there is a past time during which they were a judge presiding in court, then each is a *one-time* judge for whom there is a *past* presiding-time. Thus, a sentence with a DP whose complement is temporally independent will be true in R4-verifying scenarios, which verify something strictly stronger.

What we need, then, is to construct sentences with DPs for which this property of *temporal independence* is unavailable. Musan (1995, 1999) famously argued that the post-copular NP in an existential *there*-sentence is generally not afforded temporal independence.[29] This is illustrated in (61), where the (i) and (ii)-examples both contain DPs whose NP complements are incompatible with the rest of the utterance, but only the (i)-sentences, where those DPs are in matrix subject positions, are felicitous, contrasting with the infelicitous (ii)-sentences, where those DPs are the post-copular pivot of a *there*-sentence.

(61) a. i. Three teenagers turned two in the eighties.

ii. # There were three teenagers turning two in the eighties.

b. i. Many college sophomores were freshmen just a little while ago.

ii. # There were many college sophomores in the freshman year just a little while ago.

(adapted from O'Leary 2022)

There is some controversy in the literature as to whether being in the post-copular slot of a *there*-construction is *sufficient* for a DP to lack temporal independence. Building on work by Tonhauser (2002) arguing that this is not the case, O'Leary (2022) recently argued that only a particular subset of nouns systematically complies with Musan's generalization. While O'Leary does not provide a categorical classification, her intuition is that a noun N satisfies the generalization only if it has some obviously salient nominal alternatives, which together can be ordered on a scale $<\mathrm{N}_1, \mathrm{N}_2, \ldots, \mathrm{N}_n>$ such that, for any $i, j < n$, if a noun N_i holds of an individual x at a time t, this entails that, for any $j < i$, every noun N_j held of x at some time before t. Thus, "*teenager*" forms a scale that has this property with nouns like "*infant*", "*child*", and "*adult*", and

[29] This is a somewhat simplistic formulation of Musan's view, which rules out temporal independence for DPs in *there*-sentences *indirectly*, by restricting it to DPs with a particular discourse status, "hearer-new", and assuming (with Lumsden 1988; McNally 1992) that such DPs are disallowed in *there*-sentence post-copular positions.

"*sophomores*" similarly forms such a scale with, e.g., "*freshmen*", "*juniors*", and "*seniors*".

To convincingly show that there are attested cases of R4 relative to tense that are not due simply to the temporal independence of the nominal predicates involved, then, we must construct sentences that are verified exclusively in R4-verifying scenarios, and whose relevant nominal predicates cannot be temporally independent. Szabó, who to the best of my knowledge was the first to explore temporal R4, demonstrates this reading with the *there*-sentence in (62b), which is only true in its respective scenario if it conveys that for each of fifteen individuals there is a past time point during which they were a judge presiding in court.

(62) a. **Scenario:** While touring the grounds of a seventeenth century courthouse, no longer in use, the tour guide says:

b. There were fifteen judges who presided in this court.

However, to cover all our bases, let us also restrict ourselves to *there*-sentences with nominals like "*teenager*" or "*sophomore*" in their post-copular slots.[30]

For instance, consider the following variation on Szabó's example.

(63) a. **Context:** We are touring the grounds of an elite camp that was shut down long ago; so long that none of its former attendees are currently teenagers (perhaps none are even alive). Since the camp catered primarily to elite families, never in its existence did it host more than one or two campers from the public education system at any given time. However, throughout its existence, the camp did

[30] Another potential confound in Szabó's example will need to be avoided below: In (62b), the post-copular DP embeds a relative clause with its own past tense, headed by the noun "*judges*". However, it is known independently that the head noun of a relative clause can exhibit "reconstruction effects", behaving as if it is interpreted inside the relative clause. One might therefore wonder whether—even if the noun "*judge*" turns out to be barred from exhibiting temporal independence—the sentence in (62b) could be true in its exclusively-R4-verifying scenario in virtue of reconstruction: If the noun reconstructs into the relative clause embedded under the quantificational determiner, the noun will receive opaque evaluation relative to the embedded past tense operator, but the determiner will outscope that operator, effectively giving rise to R4 relative to embedded tense. To rule this out, we will ensure below that the predicate within the relative clause is temporally incompatible with the head noun, so that reconstruction—which triggers evaluation of the head noun and embedded predicate under the same (embedded) tense operator—will be ruled out because it results in contradictory truth conditions. In the previous chapter and in Benbaji (2021); Benbaji-Elhadad (2026b), I show that exceptions to the generalization that R4 is unattested in the modal domain—themselves due to Szabó, as mentioned in footnotes 3 and 5—always involve reconstruction of a relative head into a position below an embedded modal, parallel to what is discussed in this note with respect to tense.

admit a total of eleven teenagers who came from the public education system. To stress the snobbish nature of the camp, the tour guide says:

b. In this camp, there were (only) eleven teenagers who attended public kindergarten.

c. In this camp, there were (only) eleven individuals such that for each there is a past time interval during which they were a teenager, and a time before that, during which they attended a public kindergarten.

The sentence in (63b) seems true in the scenario spelled-out in (63a), which verifies only its R4: There are no *current* teenagers in the camp—ruling out its *de re* and *third* readings relative to matrix past tense—and no single past interval during which there were eleven teenagers who attended public kindergarten in the camp—ruling out its *de dicto* reading. The sentence is only true in the given scenario if it can convey what the paraphrase in (63c) does; namely, its R4.

The plan for the remainder of this chapter is to illustrate how we can derive the true reading of (63b) in scenario (63a) as a cumulative reading. Since, once certain simplifying syntactic assumptions are made explicit, the derivation follows quite closely our derivations of the unattested R4s in the modal domain above, for brevity, I will only provide a derivation of the R4 of (63b) in a framework that posits time variables at LF. I will then explain why the cumulative analysis must be the one with which we account for such temporal cases of R4, or—at the very least—must not be independently blocked as an account of temporal R4. Finally, I will conclude by discussing the implications of this result for our theory of intensionality: In particular, I highlight the fact that the two conclusions advanced in this chapter—namely, that we must block the derivation of R4 relative to modals via cumulation, and that R4 relative to tense must be derived via cumulation—require that we adopt a framework that places distinct constraints on worlds and times.

3.7.2 A cumulative analysis of R4 relative to tense

I adopt a framework in which time-dependent lexical items denote *temporal intensions* of type $\langle i, et\rangle$; i.e., functions that take a time-argument of type i, and return a predicate of type $\langle e, t\rangle$. This time argument—like the world argument in the world variable framework—is saturated at LF by an object-language variable over times, which is bound by one of several λ-binders, posited under each tense operator and at the matrix level.

For simplicity, I assume that present tense is semantically vacuous (65a), and that past existentially quantifies over intervals (65b), maintaining the parallelism with existential modals. With (e.g.,) von Fintel & Heim (2011), I take quantification to be restricted to a *particular* interval set, which is either contextually supplied, or overtly expressed by a temporal adverbial; e.g., (66). A past tense utterance is then true if the proposition under tense is true in a past interval in the restrictor.

(64) a. $[\![\text{teenagers}]\!] = \lambda t \lambda x.\ x$ is a teenager at t (shorthand, $\text{teenager}(t, x)$)

b. $[\![\text{attend public kindergarten}]\!] = \lambda t \lambda x.\ x$ attends public kindergarten at t (shorthand, $\text{kindergarten}(t, x)$)

(65) a. $[\![\text{PRES}]\!]^g = \lambda p_{\langle i,t\rangle} \lambda t_i.\ p(t) = 1$

b. $[\![\text{PST}]\!]^g = \lambda q_{\langle i,t\rangle} \lambda p_{\langle i,t\rangle} \lambda t_i : \exists t' < t[q(t')].\ \exists t'' < t : q(t'') \wedge p(t'')$

(66) a. $[\![\text{in the 1920s}]\!]^g = \lambda t_i.\ t$ is in the 1620s $= \{t \mid t \text{ is in the 1920s}\}$

b. $[\![\text{PST [in the 1620s]}]\!]^g = \lambda p_{\langle i,t\rangle} \lambda t_i : \exists t' < t\ [\![\text{in the 1920s}]\!]^g(t').\ \exists t'' < t :$ $[\![\text{in the 1920s}]\!]^g(t'') \wedge p(t'')$

The example in (63b) takes the form of a *there*-sentence (Milsark, 1974) with a past tense copula, and a post-copular position occupied by a DP embedding a relative clause that is itself in the past tense.[31] Since the focus of the discussion is not the syntax and semantics of *there*-sentences, I will assume for ease of exposition, a naive toy analysis of such sentences, in which the post-copular DP has vacuous scope, and "*there*" and the copula in the sentence are themselves semantically vacuous as well, with tense on the copula making the only semantic contribution beyond the DP. This is spell-out in (67), where the determiner's scope argument is saturated by an identity function.[32]

(67) [There BE+tense [[$_{\text{DP}}$ D [NP]][$\lambda x.x = x$]]]

[31] Given that we are concerned with the temporal properties of, e.g., (63b), in which a past-tense relative clause is embedded under matrix past, a disclaimer about *past-under-past* constructions is in order: These are known to give rise to "later-than-matrix" readings, on which the embedded eventuality occurs after the matrix one (Kamp, 1971). Keshet (2010b) in particular, illustrates that a relative clause internal predicate may get evaluated *after* the NP heading the relative clause, as long as both are evaluated *before* utterance time. Below, I ignore such cases, focusing exclusively on readings that chronologically order the embedded eventuality before the matrix one. I do so not because "later-than-matrix" readings are at odds with anything that will be proposed here, but rather, because their account requires further exposition that will take us too far from our main point. Our ultimate account of R4 in the temporal domain remains unchanged, even given the availability of later-than-matrix readings.

[32] As far as I can tell, nothing essential rests on this toy implementation.

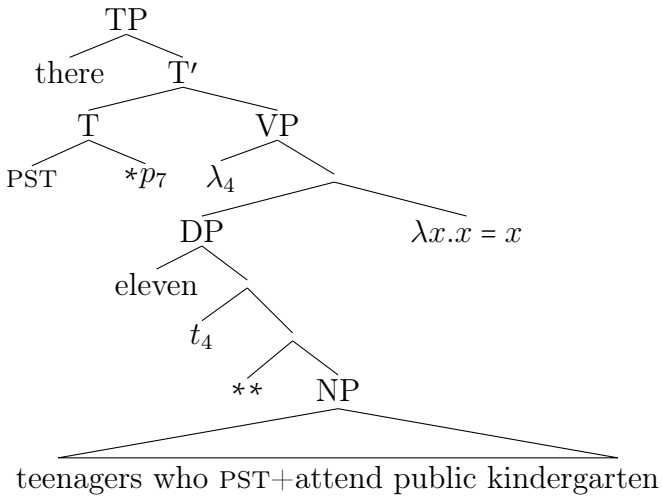


Figure 3.6: LF that derives the R4 of (63b)

With these assumptions in place, we can now illustrate how R4 is derived for (63b) via cumulation, given a structure along the lines of Figure 3.6. The NP in the Figure, a relation between times and individuals (68a), is closed under sum-formation by the cumulation operator $**$, deriving the relation in (68b), which holds (also) between pluralities of times and individuals. The DP is then assigned the generalized quantifier denotation in (68c), and given that its scope is assumed to be vacuous, we derive the temporal proposition in (68d) as the denotation of VP.

(68) a. $[\![\text{NP}]\!]^g = \lambda t \lambda x.\ \text{teenager}(t, x)\ \wedge \exists t' < t : \text{kindergarten}(t', x)$

b. $[\![**\text{NP}]\!]^g = \lambda t \lambda x.$
$[\forall y \leqslant x : \exists t' \leqslant t : \text{teenager}(t', y)\ \wedge\ \exists t'' < t' : \text{kindergarten}(t'', y)] \wedge$
$[\forall t' \leqslant t : \exists y \leqslant x : \text{teenager}(t', y)\ \wedge\ \exists t'' < t' : \text{kindergarten}(t'', y)]$

c. $[\![\text{DP}]\!]^g = \lambda f_{\langle e,t\rangle}.\ \exists x : |x| = 11\ \wedge\ f(x)\ \wedge\ [\forall y \leqslant x : \exists t' \leqslant t_4 :$
$\text{teenager}(t', y)\ \wedge\ \exists t'' < t' : \text{kindergarten}(t'', y)]\ \wedge\ [\forall t' \leqslant t_4 : \exists y \leqslant$
$x : \text{teenager}(t', y)\ \wedge\ \exists t'' < t' : \text{kindergarten}(t'', y)]$

d. $[\![\text{VP}]\!]^g = \lambda t.\ \exists x : |x| = 11 \wedge$
$[\forall y \leqslant x : \exists t' \leqslant t : \text{teenager}(t', y) \wedge \exists t'' < t' : \text{kindergarten}(t'', y)] \wedge$
$[\forall t' \leqslant t : \exists y \leqslant x : \text{teenager}(t', y) \wedge \exists t'' < t' : \text{kindergarten}(t'', y)]$

The final meaningful step in the derivation combines this proposition with matrix past tense, whose restrictor—the contextually supplied set of intervals, contributed by p_7 in Figure 3.6—is itself closed under sum-formation with the one-place cumulation operator $*$. The entire sentence thus denotes the

temporal proposition in (69), which is true of a plurality of past time intervals just in case there is a plurality of eleven individuals, each of whom has an interval in the time plurality in which they were a teenager who previously attended public kindergarten.[33]

(69) $[\![\text{TP}]\!]^g = \lambda t : \exists t' < t \in g(7).\ \exists t' < t \in g(7) : \exists x : |x| = 11 \ \wedge$
$[\forall y \leqslant x : \exists t'' \leqslant t' : \text{teenager}(t'', y) \ \wedge \ \exists t''' < t'' : \text{kindergarten}(t''', y)] \ \wedge$
$[\forall t'' \leqslant t' : \exists y \leqslant x : \text{teenager}(t'', y) \ \wedge \ \exists t''' < t'' : \text{kindergarten}(t''', y)]$

Of course, this proposition holds in the scenario described in (63a), which verifies exclusively the R4 of (63b). In that scenario, it is stipulated that throughout the camp's existence, eleven individuals who previously attended public kindergarten were admitted (as teenagers) to the camp. Thus, there is a plurality of eleven past time intervals, each corresponding to a time during which one of these eleven individuals were teenagers at the camp. We thus derive a true reading for (63b) in the context of (63a), using cumulation of temporal intensions. Next, I will attempt to show that (at least in some cases) we *must* resort to cumulation of temporal intensions to derive this reading.

Now, our example controls for the absence of temporal independence of the DP "*eleven teenagers*" both in that this DP is in the post-copular position of a *there*-sentence and in that its restrictor fits the profile of nominal predicates that generally fail to exhibit temporal independence. So the most obvious alternative derivation of the truth of (63b) in its exclusively-R4-verifying scenario (63a), which appeals to temporal independence, is blocked. There may, of course, be other alternative derivations that do not appeal to cumulation of temporal intensions, and which our discussion above did not preempt. However, even if that is the case, the analysis of (63b) in Figure 3.6 that does appeal to cumulation, should at the very least not be ruled out. That cumulation is needed here, regardless of whether there turns out to be an alternative that can derive the effects of R4 without it, can be demonstrated by adding a *plural* time-frame adverbial to an example like (63b), as in (70).

(70) During two very brief time periods in this camp's existence, in the mid 1920s and late 1930s, there were eleven teenagers in the camp who attended public kindergarten.

Without cumulation between time intervals and individuals in (70), the sentence has only a strong distributive reading, on which for each of the two

[33]The proposition also requires that for each interval in the interval plurality, there be an individual in the individual plurality who was a teenager at that interval and previously attended public kindergarten. I omit this in the text for readability, as things are already quite complicated.

time intervals contributed by the temporal-adverbial, there were eleven individuals who were teenagers in the camp (who previously attended public kindergarten). But since this sentence is also felicitous when the *total* number of teenagers who previously attended public kindergarten is eleven, cumulation is independently needed.

3.8 Conclusion

In the first part of this chapter, I showed that any theory of intensionality that allows lexical items or nodes dominating them at LF to denote modal intensions of type $\langle s, et \rangle$ overgenerates an unattested R4 of modal sentences like (71), repeated from (17), once it is paired with independently motivated mechanisms that derive cumulative inferences via LF operators. This is because these mechanisms can close modal intensions under sum-formation resulting in a reading that is true in exclusively-R4-verifying scenarios. I further show that even a theory that does *not* posit intension-denoting nodes at LF, like the Scope Theory of Intensionality (STI), can in principle generate the unattested reading if we assume that cumulation can take place in the lexicon, but point to some general considerations that may block the lexical approach to cumulation.

(71) Two victims might be trapped under the rubble.

This result generalizes Schmitt's (2023) recent finding, that certain linguistic expressions previously argued to denote pluralities of worlds do not seem to give rise to cumulative inferences. It lends further support to the claim that Schmitt defends, that we simply should not be able to form world pluralities in our semantic derivations—neither in the lexicon nor in the syntax—by showing that this new claim must hold for our theories of intensionality to be able to derive a generalization long treated by many as a virtual truism.

I then argue that R4 is attested relative to tense, and that the cumulative analysis of R4—which should be blocked in the modal cases—must be available for the temporal ones. In other words, cumulation operators should be able to take temporal intensions of type $\langle i, et \rangle$ as their input. The key example involved *there*-sentences with nominal predicates that are independently barred from exhibiting *temporal independence* from the tense of their host clause, showing that the attested reading cannot be explained away by appealing to Enç's (1982; 1986) observations about temporally-shifted nouns. Furthermore, the addition of plural time-frame adverbials confirms that the temporal case genuinely requires cumulation of temporal intensions.

Taken together, these two results require that whatever formal framework we adopt, it must block cumulation from applying to modal intensions while permitting it to apply to temporal intensions. If one finds my tentative proposal that cumulation of modal intensions should be blocked by adopting STI, augmented with certain constraints on cumulation, this would mean that we would need a different formal framework for dealing with times. Alternatively, if one opts for a world-variable framework or a split-intensionality system, a dedicated stipulation must be imposed, ensuring that sum-formation would not target modal intensions, but would freely target temporal ones.

As Schmitt (2023) already notes, this is quite surprising from the perspective of the thesis of *ontological symmetry*, defended in Schlenker 2006, according to which worlds and times should have parallel formal representations. That thesis was posited based on the observation that worlds and times pattern together in many respects. It is also in tension with some very recent findings by Meisenbichler (2025, 2026), who provides evidence that world and time indices must be bound together. Specifically, even in cases where the world and time of evaluation of a certain predicate are non-locally bound, both must be bound by the *same* non-local operator. Reconciling these different conflicting results constitutes an important open challenge for future work in intensional semantics.

Chapter 4

Constraints on questions and answers: The negative, the positive, and the probabilistic

(Joint work with Omri Doron)

4.1 Introduction

Grice (1975) famously argued that the interpretation of utterances in discourse is governed by the Maxim of Quantity (among other maxims), which is composed of two clauses:

(1) **Maxim of Quantity (MoQ):**

Q1 Make your contribution as informative as is required (for the current purposes of the exchange).

Q2 Do not make your contribution more informative than is required. (Grice 1975)

While Q1 is widely accepted as a necessary component of a theory of discourse, the status of Q2 is less clear. Grice himself hedges when introducing it, hypothesizing that this second clause might not be needed as an independent component, as being over-informative might just constitute "a waste of time" (1975: 46). This can be understood as suggesting that independent economy considerations—perhaps, say, those penalizing overly long or complex utterances—might be responsible for the apparent requirement that sentences not contribute too much information. The question of whether and how to

incorporate Q2—i.e., a restriction on over-information—in our theory of discourse has thus been a defining question in the pragmatic literature post-Grice (famously, Horn 1984; Sperber & Wilson 1986; Green 1995; Levinson 2000, among many others).

But before we can talk about *too much* information, we should first have a theory that determines which information is *just enough* information. We will adopt a framework which cashes out this notion in terms of the relations between questions and answers. The starting point for our discussion is the assumption, widely-held in the pragmatic literature, that declarative sentences in discourse are always uttered relative to an explicit or implicit question (cf. Lewis 1979; Roberts 1996/2012), and are required by some pragmatic principle to *address* that question. However, there are many ways in which a sentence can address a question, with each distinguishing between different sets of potential answers, and it is an open question which of these distinctions are in charge of regulating the relations between questions and answers.

Our main goal in this chapter is to argue that a constraint on over-information is not only at play in discourse but furthermore must be "baked into" our definition of what it means for an utterance of a declarative sentence to address a question. We will defend a particular definition, according to which a sentence addresses a question only if it does not provide unsolicited information relative to that question, thus ruling out answers that express too much information. While there are good reasons to think that additional considerations, including those appealing to structural economy, regulate the length and complexity of utterances (preventing waste of time and resources), we will argue that these cannot explain the full range of data without a constraint explicitly targeting over-information.

We distinguish, following von Fintel & Heim (2011: 152), between two kinds of constraints that have been suggested to play this role. One consists of constraints requiring utterances to provide certain information relative to the question, thus corresponding to the first clause, Q1, in Grice's MoQ (1). For instance, on Roberts's (1996/2012) view, to properly address a question, an utterance must provide information that rules out (i.e., is incompatible with) at least one of its possible answers. More recent proposals in the same vein demand that an utterance provide information that increases or decreases the probability of an answer to the question it is used to address.[1] The constraints in this class all share the underlying assumption that an utterance addresses a question only if it conveys information that helps us choose among answers to

[1]See, for instance, Büring 2003; Agha & Warstadt 2020; Feinmann 2020, 2026; Benbaji & Doron 2023; Warstadt, Agha, & Franke 2026.

the question, with the relevant notion of "helpfulness" varying in its meaning among particular members of the class.

The other family consists of constraints that *forbid* an utterance from providing certain information, thus corresponding to the second clause, Q2, of Grice's MoQ. If constraints in the former class demand that an utterance be helpful relative to a question, constraints in this latter class demand that an utterance be to the point. There is only one example of a constraint of this sort in the literature, most famously discussed by Groenendijk & Stokhof (1984); Lewis (1988a,b), according to which an utterance addresses a question only if it does *not* provide any information that was not solicited by the question. This constraint is *purely negative*, as it requires only the absence of any information that was "not specifically asked for" (von Fintel & Heim 2011: 152). We will therefore call the constraint in this second class *negative*, and contrast it with constraints in the first class, which we will call *positive*.

A note on our chosen terminology: In the literature, the negative constraint defined intuitively above is often called a *relevance* constraint. Positive constraints, on the other hand, are called *informativity* constraints by some, and *relevance* constraints by others. To avoid terminological mishaps, we use the distinction *positive* vs. *negative*, which to the best of our knowledge comes from the discussion cited above from von Fintel & Heim 2011. Later on, we will provide rigorous formal definitions of what it means for a constraint on the relation between an utterance and a question to be positive or negative. For the time being, however, we will rely on the intuitive definition, according to which positive constraints demand information that is helpful relative to a question, while the negative one prohibits information that is unsolicited given a question.

This division of the constraints on answerhood allows us to make precise our defense of the role of Q2 in our theory of discourse. We argue that the relation between declarative sentences in discourse and the (explicit or implicit) question they address is constrained in a negative way, and this entails Q2 (over-information is a particular kind of unsolicited information). This, of course, does not mean that the relation is not also constrained in a positive way, by something like Q1. But, we will argue, given the necessity of baking Q2 into our constraints on answerhood, the positive constraint required to capture the empirical landscape can be quite weak.

The position that our pragmatics imposes a negative constraint may seem like a non-starter, given the range of attested discourses in which an answer is felicitous despite providing information that is clearly unsolicited relative to an overt question that precedes it. This is exemplified in (2). All potential responses there are helpful given the question: (2a) and (2b) resolve the ques-

tion affirmatively, and (2c) provides information that makes the affirmative answer more probable than the alternative.[2] However, only (2a) refrains from providing unsolicited information; in addition to helping us out with the overt question, (2b) and (2c) also provide the unsolicited information that John has a pet hamster, or that the speaker spoke to John this morning.

(2) Is John coming to the party?

a. Yes.

b. He'll come with his pet hamster.

c. He seemed super-hyped about it when we talked this morning.

Given that all three utterances seem felicitous as responses to the overt question, suggesting that a grammatical constraint rules out two of them obviously requires further explanation. Prima facie, at least, the constraint we intend to defend undergenerates many felicitous discourses. We will therefore begin by spelling out why relying on the felicity of overt question-answer pairs is a problematic methodology when it comes to determining the underlying constraints on discourse. In particular, we show that just about any proposed constraint on discourse—positive *or* negative—suffers from an apparent undergeneration problem of the sort (2) presumably posits to our negative constraint, and necessitates an auxiliary mechanism to capture the complexity of the empirical landscape. The standard mechanism that is usually appealed to in such cases, often called *QuD-accommodation*, lets an interlocutor address a question that is distinct from the one that they were explicitly asked, so that in (2b) and (2c), the speaker is not actually addressing the question of whether John is coming to the party, but a different, implicit question, relative to which these answers provide only helpful information.

We argue that the availability of a QuD-accommodation mechanism heavily confounds the standard empirical arguments in the debate over the constraints that govern felicitous discourse: Given that all approaches appeal to such a mechanism to avoid undergeneration, without controlling for the availability of

[2]Note that (2c) can be understood in two ways. On the interpretation assumed here, it conveys—more or less—that John is probably coming to party, given that if one seems super-hyped about a party the likelihood that they will attend it is high. On another interpretation, however, (2c) is effectively an affirmative answer to the question. This understanding of the answer requires that it be taken for granted that John being super-hyped about a party contextually *entails* that he will attend it. On this understanding (2c) falls into the same category as (2b); both resolve the question while providing unsolicited information. While we do not deny that (2c) can be used to convey an affirmative answer in this way, we focus on its other interpretation here.

QuD-accommodation, apparent undergeneration cannot be used to undermine *any* approach, including the negative one. The picture that will emerge from this discussion is as follows: While felicity of an overt question-answer pair does not necessarily indicate that this pair satisfies the pragmatic constraints on question-answer pairs, as we have other tools to account for their felicity even if it does not, *in*felicity of a particular question-answer pair is harder to explain without appealing to such constraints.

We then present our core arguments in favor of a negative constraint on discourse, which is sensitive to the distinction between answers expressing only solicited or also unsolicited information. One argument is based on the observation that, unlike (2b) or (2c), not *all* helpful answers that also provide unsolicited information seem felicitous; an observation that is difficult to account for if there is no constraint that regulates the acceptability of unsolicited information, but is fairly easy to explain if there is such a constraint, given the availability of QuD-accommodation. The second argument develops a diagnostic for QuD-accommodation, and uses it to illustrate that the mechanism is active in discourses consisting of question-answer pairs in which the answer provides unsolicited information, thus violating the negative constraint we defend relative to the overtly articulated question, and—crucially—inactive when an answer provides only solicited information.

Finally, we explore the implications of our arguments on some related debates in the literature, concerning the constraints that regulate how questions should be addressed. First, should probabilistic notions be incorporated into the definitions of these constraints—as in, for instance, the positive constraint alluded to above, demanding that an utterance raise or lower the probability of an answer to the question relative to which it is uttered—or not? We will show that the necessity of a negative constraint renders moot the debate over whether positive constraints on discourse should be formulated by appealing to probabilities: probabilistic and non-probabilistic formulations of the positive constraint end up being equivalent given the necessity of the negative one. We will also show that a probabilistic reformulation of the negative constraint itself is equivalent to its non-probabilistic one.

Second, we examine an alternative *reductionist* approach, that rejects the basic framework that the competing theories we compare in the first place take for granted. On the reductionist view, due to Carlson (1982), there need not be dedicated constraints on question-answer pairs of the kind we introduce above. Rather, any discourse move is constrained by the *rationality* of discourse participants, who want to maximize utility while minimizing cost, and thus use the least effort to provide as much information that is helpful in advancing their goals. We will stress that, while our arguments for a negative

constraint on discourse do not directly target the reductionist approach, this approach is tenable only if the cost function assumed to govern the behavior of rational agents itself directly penalizes unsolicited information (i.e., has a negative constraint baked into it).

Our discussion is organized as follows: Section 4.2 introduces the formal framework within which we will be operating, and reviews the approaches that we will be comparing throughout the chapter. In Section 4.3, we discuss the argument from undergeneration against the negative approach, and illustrate why the existence of a difference between our intuitions about what constitutes felicitous discourse and what a constraint allows or rules out does not by itself constitute an argument against said constraint. Section 4.4 then provides two arguments for a negative constraint that bars unsolicited information. In Section 4.5 we turn our attention to the debate over probabilistic formulations of discourse constraints, and show that—given the necessity of the negative constraint—they are equivalent to non-probabilistic ones, rendering the introduction of probabilities superfluous. Section 4.6 introduces an alternative reductionist approach and presents its points of contact with our arguments for the negative constraint. Section 4.7 concludes.

4.2 Introducing the formal framework

The formal framework we will be assuming here is one in which utterances are always made with respect to a *common ground* and a *Question under Discussion* (QuD). The common ground is defined, following Stalnaker (1978), as a set of propositions that discourse participants take to be shared knowledge among themselves, which in turn is used to derive the conversational *context set* c in (3); namely the set of worlds compatible with the common ground. The QuD, as defined in (4) following Roberts (1996/2012), is a *partition* of this set of worlds into disjoint, non-empty subsets, and serves to define the task that discourse participants are faced with at a given point of the conversation; namely, the task of determining which, among the cells of the QuD, is home to the actual world.

(3) $c = \bigcap\{p \mid p \text{ is shared knowledge among discourse participants}\} =$
$\{w \mid w \text{ is compatible with the shared knowledge of discourse participants}\}$

(4) A model of discourse is a pair $\langle c, \mathcal{Q} \rangle$, such that:

a. c is the discourse *context set* (as defined in (3)).

b. $\mathcal{Q}$, the *QuD*, is a *partition* over c; i.e., a set of propositions $\{q_1, ..., q_n\}$ such that:

i. $c = q_1 \cup q_2 \cup ... \cup q_n$

ii. $\forall i \leq n \; [q_i \neq \varnothing]$

iii. $\forall i, j \leq n \; [i \neq j \rightarrow q_i \cap q_j = \varnothing]$

The QuD may correspond to an overtly-asked question, but (again after Roberts 1996/2012) we assume that a QuD is present in the background of any utterance, even in the absence of any explicit question. As we will see, the need to coordinate the QuD at any point in discourse among its participants will play an important role in the phenomena we examine in this chapter.

With this in place, let us spell out one distinction that clearly plays a role in constraining discourse, between utterances that are *informative* relative to the context set, and those that are not. That some constraint is in place ruling out instances of the latter is evident from the infelicity of, say, (5). This constraint is formulated by Stalnaker (1978) as one preventing speakers from uttering what is already entailed by the context set (6).[3]

(5) As Jane and Sue watch TV, Jane suddenly says: #*The TV is on.*

(6) **Informativity (INF):** An utterance of a declarative sentence ϕ is felicitous given c only if $[\![\phi]\!] \cap c \neq c$.

There is, however, another form of infelicity—exemplified in (7)—that cannot be attributed to INF but which nevertheless seems to involve a violation of some rules governing how agents should behave in discourse. The general intuition *vis-a-vis* (7) is that the infelicity of Jane's utterance is due to its failure to address the question being discussed; Sue and Mira are talking about football, as Jane brings up her car problems out of the blue. Since, in the framework assumed here, a QuD is present even in the absence of an overt

[3] Jane's utterance in (5) may not be completely unacceptable. It might be quite natural if, for example, Jane is under the impression that Sue got lost in thought and forgot that the TV was on. Notice, however, that this scenario makes the proposition that the TV is on not common ground. It is in fact very difficult to construct an example which is conclusively infelicitous, due to the difficulty of pinning down the common ground in a given scenario. But this is by no means evidence against the informativity constraint formulated in (6). On the contrary, the fact that the manipulations that can salvage Jane's utterance in (5) are exactly the ones which imply that it is not entailed by the context set should be viewed as evidence supporting this constraint.

question, infelicities like Jane's utterance in (7) could be modeled as a violation of a constraint on *question-answer pairs* requiring that every declarative utterance in discourse address the implicit or explicit question relative to which it was uttered (Collingwood 1940/2001; Carlson 1982; Groenendijk & Stokhof 1984; Lewis 1988a; Roberts 1996/2012; Farkas & Bruce 2010 a.m.o.). If in the case of (7) the implicit QuD is assumed to be *was there a foul?* then Jane's utterance is infelicitous because it fails to bear on that question.[4]

(7) **Context:** Jane, Mira, and Sue argue about a football match that just ended. Mira and Sue have not seen Jane in a while, and do not know anything that happened to her lately.

a.Sue: The referee should not have called that foul.

b.Mira: What are you talking about! It was an obvious foul!

c.Jane: #My car is blue.

Now all that is left for us to figure out is what exactly a constraint demanding that an utterance address a question consists of. In the early days of formal pragmatics, two answers to that question co-existed in the literature, which, as noted we will call *negative* and *positive*.

One early position, inspired by Groenendijk & Stokhof's (1984) work on question-answer pairs and by Lewis's (1988a; 1988b) work on the notion of being *about a subject matter*, suggests that the constraint distinguishes utterances which provide information unsolicited given the QuD from those utterances that provide only solicited information, with only the latter counting as addressing the question. This is spelled out as a formal constraint in (8), which we call the *negative Question-Answer constraint*, or NEG in short, demanding that the meaning of an utterance must either be entailed by or contradict every cell in the QuD.[5]

[4]Again, Jane's sentence here is not always unnatural. As discussed by Grice (1975), by uttering a completely unrelated sentence, Jane might be signaling that she wishes to change the topic of conversation. In other words, Jane's utterance might be intended to address a different question than the one addressed by Sue and Mira's utterances. To the extent that Sue and Mira accept this question as the new QuD, Jane's utterance can be judged felicitous. Again, we see that the difficulty of pinning down the QuD makes it hard to construct an unambiguously infelicitous example; and again, we suggest that the profile of the manipulations that can be used to salvage Jane's utterance in (7) should be viewed as further evidence for the existence of a constraint in discourse related to the QuD.

[5]By *entail* and *contradict* we mean *contextually entail* and *contextually contradict*; namely, entail/contradict when intersected with the context set. The relations between meanings relevant to the discussion here are always contextual, and so throughout this chapter we will often drop the word *contextual* for the sake of brevity.

(8) **Negative Question-Answer constraint (NEG):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q} \rangle$ only if:
$\forall q \in \mathcal{Q}$: $q \subseteq [\![\phi]\!] \vee q \cap [\![\phi]\!] = \varnothing$.

The other approach in the early literature, due to Roberts (1996/2012), takes the basic distinction underlying our Question-Answer constraint to be the one distinguishing utterances that (at least partially) resolve the QuD from those that do not. This notion, which we term the *positive Question-Answer constraint* (POS, in short) in (9), demands that an utterance express a proposition that *entails* a proper subset of the QuD; i.e., eliminates at least one of its cells.

(9) **Positive Question-Answer constraint (POS):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q} \rangle$ only if:
$\exists q \in \mathcal{Q}$: $q \cap [\![\phi]\!] = \varnothing$.

Notice that NEG does not rule out examples like (5) above, which are entailed by the context set; namely, entailed by every cell in the QuD. NEG is thus not a generalization of INF; to account for the full range of infelicitous examples, utterances should be subject to both constraints. While we can lump the two together to form the constraint in (10) (sometimes termed *strong relevance* in the literature; e.g., Križ & Spector 2021; Haslinger 2023), which requires utterances to express propositions that are equivalent to a proper subset of the QuD, this is a mere convenience for presentational purposes. If we think that INF is independently required, NEG isolates what is required on top of it: While INF informs a speaker of what she must *do*, namely, ensure that worlds are eliminated from the context set, NEG instructs her what *not* to do, i.e., ensures that she does not harm the integrity of cells in the QuD in the process of world-elimination.

This serves to highlight another important point. In the introduction we define the notions of positive and negative constraints relative to an utterance and a *question*: a constraint is positive if it demands that an utterance provide certain information that is helpful given the question, and negative if it demands that the utterance refrain from providing information that is unhelpful. However, there is a somewhat simpler—and perhaps more intuitive—definition, according to which positive constraints simply demand that an utterance provide information (and can define which kind of information, perhaps in relation to a question), whereas negative ones demand that an utterance refrain from providing information (and can, again, define which kind of information in relation to a question). In this sense, INF—which demands non-triviality—is a positive constraint, while NEG is a negative one.

(10) **INF+NEG:** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q} \rangle$ only if $[\forall q \in \mathcal{Q}\colon q \subseteq [\![\phi]\!] \vee q \cap [\![\phi]\!] = \varnothing] \wedge [\![\phi]\!] \cap c \neq c$.

On the other hand, unlike NEG, POS *is* a generalization of INF: If INF requires that a speaker rule out some possibilities with their utterance, then POS reduces to INF with a coarser-grained distinction among possibilities; now construed as partition cells rather than as particular worlds. In other words, while an utterance can satisfy INF without satisfying NEG, and *vice versa*, POS renders INF superfluous: All declarative utterances ruled out by the latter, i.e., those that do not eliminate *worlds*, are trivially ruled out by the former, as they also do not eliminate *cells*.

This means that INF+NEG in (10) results in a constraint that is equivalent to the one that would result from pairing NEG and POS. We flag this here because we will immediately argue that discourse is constrained by NEG together with an informativity constraint. Now, given that INF+NEG is equivalent to POS+NEG, our argument will not be an argument against POS. For all we know, POS might constitute a grammatical constraint in addition to NEG (this, in fact, seems quite plausible to us). Rather, we will argue that a constraint like NEG, which distinguishes between solicited and unsolicited information in an answer, is required for explaining the facts.

Notably, if an overt question is assumed to be the QuD that a succeeding answer must address (an assumption that we will scrutinize in the next section), both POS and NEG undergenerate a wide range of utterances. For instance, only one of the answers to (2), repeated in (11), satisfies NEG, and only two satisfy POS: (11a) satisfies both constraints, ruling out a negative answer to the question while not conveying any further information, while (11b) conveys *too much information* to satisfy NEG, but still satisfies POS; it not only rules out the possibility that John will not come, but also distinguishes worlds in which he comes with his pet hamster from the remaining worlds in which he comes without it. In (11c)-(11d) the speaker is conveying *too little information* to satisfy either constraint; all these answers can be true or false regardless of whether John will, in fact, attend the party, and so by definition fail to eliminate any cells of the partition, while also distinguishing among worlds in a way that cuts across cells.

(11) Is John coming to the party?

a. Yes

b. He'll come with his pet hamster.

c. Probably.

d. He seemed super-hyped about it when we talked this morning.

The reason (11c)-(11d) seem acceptable as responses to the question is that, pre-theoretically, they provide us with information that renders one answer more plausible. Granted, that John will probably attend the party or that he sounded super-hyped about it earlier does not settle the question of whether he will attend; both leave open the possibility that he will not. However, both answers push us towards an affirmative answer to the question.

In the informal discussion above, we appealed to probabilistic language, to terms like "likely" and "plausible", to explain our intuitions regarding (11c)-(11d). Thus, to capture this intuition, more recent formalizations of Question-Answer constraints introduce probabilities into the definition.[6] The various technical implementations of this idea all introduce a novel parameter into our model of discourse: a probability distribution P mapping subsets of the logical space, i.e., propositions, to values between 0 and 1. An utterance ϕ is then not required to eliminate cells in $\mathcal{Q}$ to count as addressing it, but rather, *updating* ϕ must "alter" P in a particular way, on which there is some variation among the different approaches.[7] For instance, on one version of the probabilistic view, ϕ addresses $\mathcal{Q}$ given P only if $[\![\phi]\!]$ "shifts the probabilistic weights among the propositions denoted by $\mathcal{Q}$" (Büring 2003: 517).[8] This is executed in (12) by demanding that there be at least one cell in the QuD whose probability shifts once $[\![\phi]\!]$ is updated in the context.[9]

(12) **Probabilistic positive Question-Answer constraint (p-POS):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q}, P\rangle$ only if $\exists q \in \mathcal{Q} : P(q|c \cap [\![\phi]\!]) \neq P(q|c)$.

[6]The inspiration to this approach comes from Carnap (1950), who in the context of scientific theorizing, famously defined relevance of information i to an entertained hypothesis h, as the extent to which i raises our "degree of confirmation" of h. Modern probabilistic views all build on this definition, suggesting that a proposition p addresses the question of "*whether q?*" if knowing that p would shift the probability we assign in our epistemic model to q in a significant way.

[7]Cf. Warstadt & Agha 2022; Warstadt et al. 2026 for a useful overview of the different approaches.

[8]For more formal renditions of the approach, see Hawkins & Goodman 2017; Feinmann 2020. This formulation is often the one used in the linguistic literature; e.g., Hyska 2015; Benbaji & Doron 2023.

[9]An alternative proposal by Van Rooy (2004) that we set aside here, uses the notion of *entropy reduction* in defining what it means for an answer to address a question, a measure of the degree to which the answer decreases uncertainty *vis-a-vis* the resolution of the question. Russell (2012) dispenses with a categoric definition altogether, in favor of a gradient one according to which the *degree* to which ϕ addresses $\mathcal{Q}$ is the difference between the probability P assigns to an answer $q \in \mathcal{Q}$ before the context is update with $[\![\phi]\!]$ and the measure assigned to q after update has occurred (i.e., $P(q|c) - P(q|c \cap [\![\phi]\!])$), resulting in some utterances addressing $\mathcal{Q}$ "more than" others.

We call this constraint *probabilistic positive* (p-POS) because, while it is more permissive than non-probabilistic POS in that it considers non-resolving answers like (11c)-(11d) to still address their preceding question, it shares a crucial property with POS: it demands the introduction of certain information. We will further discuss this feature of p-POS, and explore whether we can define useful *negative* probabilistic discourse constraints, below. For now, however, we have enough background to turn to the main point of the chapter; namely, the defense of the negative constraint.

4.3 Evaluating the case against NEG

The three main constraints on question-answer pairs on the market, NEG (8), POS (9), and p-POS (12), each take a different distinction among answers as central to determining what counts as addressing a question. According to NEG, the critical distinction is between answers that provide unsolicited information—modeled in the framework we are assuming here as eliminating some but not all worlds in a partition cell—and those that do not. According to POS, at least partially resolving a question—i.e., being incompatible with at least one cell—is what matters. Finally, according to the probabilistic version of POS—p-POS—the critical distinction is between answers that bias us towards or against some answer to the QuD and those that do not. To model that distinction, one must enrich the model with a probability parameter, representing the probability distribution of answers to the QuD.

As long as we assume that a constraint like INF is independently needed, then the three constraints are ordered by strength in the following sense: Any answer that (is informative and) does not contribute unsolicited information relative to some QuD $\mathcal{Q}$ (satisfying NEG), also (at least partially) resolves $\mathcal{Q}$ (satisfying POS), and any answer that resolves $\mathcal{Q}$ also biases us for or against an answer to $\mathcal{Q}$ (satisfying p-POS). Our working example in (11), repeated again in (13) for convenience, is a case in point: While all answers satisfy p-POS, only the first two satisfy POS, and only the first one satisfies NEG. We thus have a hierarchy of constraints as spelled out in (14).

(13) Is John coming to the party?

a. Yes.

b. He'll come with his pet hamster.

c. Probably.

d. He seemed super-hyped about it when we talked this morning.

(14) Given a model of discourse containing at least a context c and QuD $\mathcal{Q}$, and on the assumption that any declarative utterance must be informative, $\{p|\ p$ satisfies NEG relative to $\mathcal{Q}\} \subset$
$\{p|\ p$ satisfies POS relative to $\mathcal{Q}\} \subset$
$\{p|\ p$ satisfies p-POS relative to $\mathcal{Q}\}$

The convergence of much of the recent literature on probabilistic formulations like p-POS is largely motivated by the fact that all answers in (13) seem felicitous as responses to the overt question, which is taken as evidence that the weakest constraint is required to avoid undergeneration. This reasoning is evident, for instance, in the following passages:

> I will resort to the rather vague formulation that *A is an answer to Q* if *A shifts the probabilistic weights among the propositions denoted by Q*. Stricter definitions such as in Roberts (1996, p. 104), which requires an assertive move to entail at least a partial answer to the QUD, exclude cases like (i).
>
> (i) Q: Will you come to the party?
> A: Presumably.
>
> (i) entails neither a positive nor a negative answer to Q, even though it strongly shifts the probability to the positive one and should certainly count as a relevant answer.
>
> (Büring 2003: 517)

> [Roberts'] definition [...] is overly restrictive and should be weakened at least to allow for discourse moves which merely raise or lower the probability of some answer to the QuD being correct. Consider for example the sequence: Q: "Is it going to rain?" A: "It's cloudy." A's utterance does not contextually entail an answer to the QuD (at least not in Pittsburgh, PA). Intuitively, it is relevant because it somewhat raises the probability of an affirmative answer...
>
> (Simons, Tonhauser, Beaver, & Roberts 2010: 316-317)

The general methodology suggested here goes as follows: When one encounters a question-answer pair in which the answer does not seem to address the question given a particular formulation of the constraint demanding that it do so, but that nevertheless seems felicitous, one should weaken the constraint in order to admit it. And since there are question-answer pairs that satisfy only

the weakest formulation in the hierarchy—namely, a version of p-POS—one ends up with that formulation as *the* correct one.

But is p-POS weak *enough* to admit all felicitous question-answer pairs? Unfortunately, the same kind of counterexamples that have been used to target stronger formulations, can be hurled at this one as well. Consider, for instance, examples (15) and (16). The answers in (15) and (16) obviously cut across cells in the partition imposed by the questions they succeed and therefore do not satisfy NEG with respect to it. They also do not eliminate any of its cells; the answers are compatible with both cells in the partition, and therefore do not satisfy POS relative to it either. Finally, they also do not seem to shift the probability we attribute to any of the cells; in (16), for example, the knowledge of where we are planning to hang out during our trip, or what the conditions are for us embarking on it to begin with, is completely orthogonal to the question of whether it will rain. Nevertheless, these seem to be felicitous answers to the overtly asked question. Thus, by the logic described above, we should now weaken our constraint on question-answer pairs even more.

(15) Is John coming to the party?

a. It's going to be a lame party regardless.

b. His parents gave me conflicting messages.

(adapted from Feinmann 2026)

(16) Is it going to rain during our trip?

a. We'll be spending most of the time indoors.

b. We'll only go if it doesn't.

If the methodology behind the weakening of our discourse constraint on questions and answers is sound, then, it seems that p-POS, too, must be weakened. But if we do not want to find ourselves weakening *ad infinitum* we should, perhaps, conclude that there is something faulty in that methodology to begin with.

Of course, proponents of the weaker formulations—and of p-POS in particular—are well aware of examples like (15)-(16).[10] A common assumption, explicitly stated in Feinmann 2026, is that these cases can be accounted for *without* weakening any further the constraints that govern the relation between a QuD and a declarative utterance. Instead, we can reject the assumption that the overtly-asked question is always the QuD relative to which the answer is uttered. In fact, there is a good conceptual reason to doubt this assumption:

[10] See Feinmann 2026 for a recent discussion.

Since we assume that a QuD is present even in the absence of an overtly-articulated question, we must already posit a process by which discourse participants accommodate an implicit QuD relative to which discourse is held. But then there is no obvious reason to prevent this process, whatever it is, from playing a role even when a question *was* explicitly articulated (Beaver & Clark, 2009; Roberts, 1996/2012).

Indeed, the availability of presupposition accommodation—i.e., the process by which certain information is introduced into the context set simply in virtue of a speaker's indication that she is taking it for granted—teaches us that a similar process is possible for the common ground (Lewis, 1979; Stalnaker, 1977), and given the inherent uncertainty in discourse about the topic of conversation, there is no reason to expect it not to be available for QuDs as well. In the case of, say, (16), this means that the addressee is required to accommodate a QuD along the lines of *where will we spend our time on this trip?* or *will the trip take place rain or shine?* as relative to these accommodated QuDs, the answers in (16a)-(16b) address the preceding question even according to the strictest definition in NEG.

But if a mechanism of QuD-accommodation is available in principle, this considerably complicates what kind of evidence can be appealed to in the debate over the correct formulation of our discourse constraints. In particular, the judgment that a certain overt question-answer pair seems *felicitous* can only be used as an argument against a formulation of our constraint on such pairs which appears to undergenerate it if we are able to rule out the possibility that felicity is due to QuD-accommodation in the first place.[11] More generally, as noted by Beaver & Clark (2009: 39), any formulation of discourse constraints can be fully predictive only if the possibility of accommodation is constrained. Any argument for one position or other is unsound if it relies solely on felicitous

[11] Agha & Warstadt (2020) suggest that non-sentential "fragment" answers to overtly asked questions (in the sense of Merchant 2005) block accommodation of an implicit QuD and must address the overt question that they follow. The argument is based on an observation by Jacobson (2016), according to which fragment answers to *which*-NP questions must denote elements in the extension of NP, while corresponding full sentential answers are not required to dominate expressions denoting such elements. Based on this diagnostic, Agha & Warstadt attempt to delineate a set of question-answer pairs that can be taken to bear directly on the formulation of the constraints governing how questions are addressed, given the impossibility of accommodation. This is, indeed, the general methodology we think one should follow when attempting to reason about these things, but we think it is misapplied in this case; in work under preparation, we examine the status of fragments and show that Jacobson's data cannot be used to motivate a ban on accommodation. It should also be noted that Jacobson herself provides a semantic account of the facts that has nothing to do with accommodation, or lack thereof.

discourses containing an overt question-answer pair, suggesting that the QuD cannot be shifted—contrary to fact.

As far as we are aware, the main argument in the literature in favor of positing only a positive constraint (POS and p-POS) does rely solely on felicitous discourses of this kind. But given the availability of QuD-accommodation, positing a negative constraint (NEG) can just as well predict the felicity of such discourses. In terms of empirical coverage, then, the two approaches seem to be, at the very least, on a par.

The point we make here is not, by itself, an argument for NEG, of course. It is just an argument against a popular argument against NEG. Next, we will try to make a positive case for NEG, which illustrates that as a matter of fact, without taking into consideration the distinction between solicited and unsolicited information when evaluating whether an answer addresses a question, one is at an empirical disadvantage. Recall that, as already noted, in our case for NEG we do not argue *against* the existence of a positive constraint on discourse. Rather, we argue against the view that positive constraints can do the job of constraining discourse on their own, without a constraint prohibiting answers that contribute unsolicited information like NEG.

4.4 Arguments for NEG

4.4.1 An argument from over-informative answers

Our first argument for the necessity of NEG relies on an important difference between NEG and the competing positive formulations: Only NEG limits the logical strength of utterances in discourse. If we assume that POS or p-POS is the only constraint on the relation between utterances and the QuD, and given an utterance ϕ that satisfies POS or p-POS, any utterance entailing ϕ must also satisfy POS or p-POS. This property, which we call *downward monotonicity*, is formally defined in (17). The reason POS and p-POS have this property, intuitively, is that they only demands the speaker to contribute *enough* information to eliminate or shift the probability of a cell, but do not specify *how* the speaker should do so.

(17) A one-place relation on utterances is *downward monotone* if for any two utterances ϕ, ψ such that $[\![\phi]\!] \subseteq [\![\psi]\!]$, it holds that $F(\psi) \Rightarrow F(\phi)$

NEG, on the other hand, puts an upper bound on the strength of utterances. Consider, for example, an utterance like (13a), which is contextually equivalent to one of the cells in the QuD and therefore satisfies NEG relative

to it. Any stronger utterance, as long as it is not contextually contradictory, will necessarily violate NEG, since it eliminates some but not all of the worlds in that cell, as is the case in (13b). We have therefore arrived at a fundamental difference in empirical predictions between the two kinds of systems we compare here: a system without NEG predicts that *all else being equal*, an utterance cannot be infelicitous because it is too strong; whenever an utterance is felicitous in a given context, any utterance with a stronger meaning should be felicitous as well in that context. A system with NEG, on the other hand, predicts the inverse: in some cases, infelicity may arise as a result of a meaning which is too strong. To show that NEG is needed, then, we should search for exactly such instances of infelicity.

But before we take up this task, we need to mention two possible confounds in the methodology described above. One was already discussed at length above: The availability of QuD-accommodation could make it quite difficult to find cases of infelicity that are due to a violation of NEG. Interlocutors are mostly quite charitable to each other, and even when someone utters a sentence that violates NEG relative to the QuD her interlocutors have in mind, she can expect her interlocutors to revise their assumptions about the QuD, such that under the revised assumptions, her sentence will *not* violate NEG. It is important to note that while the availability of QUD-accommodation will make it more difficult for us to construct examples of utterances which are infelicitous in virtue of their logical strength, it cannot cast doubt on the validity of such examples as evidence for NEG.

To overcome this challenge, we will consider such pairs in scenarios in which a speaker should not expect the kind of charitability that triggers QuD-accommodation from her interlocutors, due to a power imbalance between them. In particular, we will consider conversational scenarios in which one of the interlocutors is a figure of authority, taken to be the final arbiter of which questions are and are not to be addressed in conversation. In such scenarios, one cannot operate under the assumption that their interlocutor will just be willing to revise their assumptions about the QuD, as these assumptions are privileged in dictating the terms of the conversation.

The second confound has to do with the possibility that the logical strength of utterances is, perhaps, indirectly limited by *manner* constraints, targeting the syntactic structure or the surface form of utterances in discourse. Consider, for example, the contrast between the answers in (18) below. The answer in (18b) entails the one in (18a), yet the latter is felicitous while the former is infelicitous as a response to the given question. Is this enough to show that a constraint limiting the strength of utterances is needed? No, because (18b) may be ruled out for reasons related to its length or structure. It seems

reasonable to hypothesize a pragmatic economy principle which rules out an utterance if it has an alternative with a simple structure which eliminates the same cells from the QuD. This would explain the infelicity of (18b) without appealing to NEG, as a result of competition with the simpler alternative in (18a).

(18) A: Is John coming to the party?

a. He is.

b. # He is, and I had eggs for breakfast.

We would like to abstract away from the question of which exact property of the structure or surface form of utterances is targeted by such manner principles. In other words, we will stay agnostic about what it means for an utterance to be *more complex* than another utterance. Instead, we will try to work around this confound by looking only at pairs of utterances whose structure and surface form are as close to identical as possible.

We can therefore be more nuanced about the empirical prediction of each of the approaches considered here. A system without NEG predicts that whenever an utterance is felicitous in a given scenario, any utterance with a stronger meaning *and identical in its structural complexity and length* should be felicitous as well. A system with NEG predicts that, assuming we can block accommodation of a QuD relative to which it obeys NEG, the stronger utterance should be degraded. Thus, if we can find cases in which strengthening of this sort results in infelicity, this will constitute an argument for NEG .

Let us test this empirical prediction. The examples below are all constructed in the following way. The speaker in (a), an authority figure conducting an interview, poses a question which the addressee addresses in two alternative ways: in (b), the answer intuitively contains only solicited information, while the answer in (c) is strictly stronger, containing additional information. Crucially, the two alternative answers are identical in structure and form by any reasonable metric. The fact that the (b)-utterances are felicitous while the (c)-utterances are degraded teaches us that there must be some constraint in place that puts an upper bound on the logical strength of utterances.[12]

[12]This contrast is therefore completely unexpected in a system in which the meaning of utterances is only constrained by POS. Note that if the addressee in the (c)-utterances is assumed to be unaware that their utterance entails the respective (b)-sentence, their answer becomes significantly more felicitous. This, we assume, is because they are doing their best to be helpful, and so can reasonably expect their interlocutor to engage in QuD-accommodation despite the power imbalance between them.

(19) **Scenario:** Due to a warning from the CIA about sleeper terror cells in France and Spain, the US decides that anyone entering the US will be subjected to additional security screening if they visited one of these countries in the past month. Returning from vacation, Sue presents her passport to a Customs and Border Protection agent who appears very stern and strict.

a.Agent: Which countries did you visit?

b.Sue: I visited the UK and France.

c.Sue: ??I visited London and Marseille.

(20) **Context:** A patient gets to the ER of a hospital with an animal bite. The doctor needs to know whether to administer a post-exposure treatment for rabies, which is required after a dog bite, but should not be administered after a cat bite.

a.Doctor: An animal of what species bit you?!

b.Patient: It was a dog, doctor.

c.Patient: ??It was a beagle, doctor.

(21) **Context:** A building's fire safety protocol demands that no combustible material should be kept in the storage room. Wood is generally defined as a combustible material, regardless of the type. An inspector questions one of the tenants about a closet they keep in storage, to determine whether they should be fined for violating the protocol.

a.Inspector: What is your closet made of?

b.Tenant: It's made of wood, inspector.

c.Tenant: ??It's made out of birch, inspector.

Three things are of note: First, the argument above lends support to NEG only if positive approaches are unable to posit additional constraints to avoid overgeneration of the (c)-cases above. If one can come up with a constraint that filters the (c)-examples, while admitting the (b)-cases, the argument for NEG, we concede, would be neutralized.

Second, as discussed, to the extent that the c-examples above are indeed infelicitous, it means that QuD-accommodation—while in principle always available—does not kick in to salvage the infelicitous (c)-examples, due to the different roles interlocutors play in conversation. Sue cannot expect her interlocutor in (19), for instance, to accommodate a QuD along the lines of *which cities did you visit?* relative to which her (c)-answer satisfies NEG,

simply because she is not in a position to negotiate anything with the border agent, who sets the terms of their interaction. The judgment is naturally delicate, and depends on how we fill in the details in the scenario: If, instead of a stern and serious looking agent, we imagine that the Sue is interacting with a friendly agent who starts off every interaction with small-talk about his interlocutors' trip, the over-informative answer is no longer so degraded, presumably because the agent is indicating to Sue that she can plausibly expect charitability from him. Alternatively, the overinformative c-answers may be interpreted as challenging the authority of the asker – by expecting the border control agent to accommodate a different QuD than the one explicitly asked, Sue may be provocatively indicating that she does not acknowledge the power asymmetry in the situation. This demonstrates the complexity of the process of QuD-accommodation, which makes a comprehensive theory of it so elusive. But regardless of the reason that QuD-accommodation is difficult in the (c)-examples, we note that this line of argumentation is only possible within a framework that posits NEG to begin with; positive approaches without an additional ban on over-informativity simply predict answers like the (c)-examples to be felicitous across the board.

Finally, the discussion above demonstrates an interesting property of the kind of argumentation we engage in when theorizing about the grammatical constraints regulating how questions should be addressed: While felicity of an overt question-answer pair does not necessarily indicate that this pair satisfies these grammatical constraints, given the availability of QuD-accommodation, *in*felicity of particular question-answer pairs is harder to explain without appealing to grammatical constraints. This reflects an asymmetry in the nature of QuD-accommodation itself: While in some cases a speaker is allowed to pretend as if she was asked a different question than the one which was in fact uttered, she is never *forced* to do so. QuD-accommodation can ameliorate how we judge the felicity of a particular discourse (compared to how we would judge it it in a world without QuD-accommodation), but it never deteriorates it. More concretely, a theory that posits grammatical constraints that are satisfied by a given question-answer exchange predicts that exchange to always be felicitous (unless blocked by a different consideration), whereas a theory that posits grammatical constraints that are violated by the exchange has a much weaker prediction; namely, that in some cases, that exchange might be infelicitous. For this reason, examples (19)-(21) seem to us to favor the latter kind of theory.

4.4.2 An argument from diagnostics for QuD-accommodation

As we have seen above, when it comes to discourses that seems felicitous, the debate between NEG, POS and p-POS, is just about what accounts for the felicity; is it the fact that the discourse consists of a question-answer exchange that satisfies our formal constraint on such discourses, or the fact that even though the explicit exchange violates the constraint, we can accommodate an implicit QuD relative to which it does not? If we are able to develop some diagnostics that indicate when accommodation has been appealed to, we should therefore be able to tease apart the proposed approaches on empirical grounds. In particular, if we are correct that NEG indeed regulates the relationship between utterances and the QuD, then we would expect that any felicitous discourse in which an answer does not satisfy NEG relative to an overt question, will present hallmarks of accommodation—to the extent that these can be diagnosed.

It is a general property of discourse that not every expectation that something be accommodated must be respected (Szabó, 2006; von Fintel, 2008). In the domain of presuppositions, this is illustrated by the observation in Shanon (1976), termed the *Hey, wait a minute!* observation by von Fintel (2004), that an addressee may "legitimately complain" when the speaker presupposed something not previously taken to be an established fact. This diagnostic sets apart presupposed and asserted content; the addressee's complaint cannot target a speaker's assertion, as illustrated in (22).

(22) A: The mathematician who proved Goldbach's Conjecture is a woman.
B: Hey, wait a minute! I had no idea that someone proved it.
B′: #Hey, wait a minute! I had no idea it was a woman.
(von Fintel, 2004)

We can develop an equivalent test for accommodation of QuDs. More specifically, the idea is that, when a speaker responds to an explicitly articulated question $\mathcal{Q}$ using an utterance that does not satisfy NEG relative to $\mathcal{Q}$, their addressee may indicate their refusal to engage in accommodation by uttering, say, *I didn't ask you* $\mathcal{Q}'$*, I asked you* $\mathcal{Q}$*!*, where $\mathcal{Q}'$ is the question that should be accommodated, relative to which the utterance *does* satisfy NEG. We call this the *I Didn't Ask You*, or IDAY, test.[13] Take, for instance, the discourses in (23)-(24), with which we used to exemplify answers that do not satisfy any of the constraints in the literature relative to the questions that overtly precede them. These are cases for which proponents of any theory—including p-POS—must concede that accommodation is required.

[13]We thank Danny Fox (p.c.) for suggesting this to us.

(23) A: Is John coming to the party?
B: It's going to be a lame party regardless.
A: I didn't ask you what the party will be like, I (only) asked you whether John is coming!

(24) A: Is it going to rain during our trip?
B: We'll be spending most of our time indoors.
A: I didn't ask you where we'll be spending our time, I (only) asked you whether it is going to rain!

Since B expects A to accommodate a different question, we expect that A should be able to resist accommodation and, in fact, A can do so.

Importantly, we can show that the only kind of answers to which IDAY seems to be an inappropriate response are precisely those that satisfy NEG relative to the overt question that precedes them. For instance, consider (25), where B's answer satisfies NEG relative to A's question, but only partially resolves it. Prima facie, one might think that A has reason to protest; she asked for more information than her interlocutor provided. That she cannot respond with IDAY in this case can be taken to support the view that, indeed, what IDAY targets is QuD-accommodation, which is not needed here as the overt discourse satisfies NEG on its own.

(25) A: Who among John, Sue and Mary is coming to the party?
B: John or Mary are.
A: #I didn't ask you whether one of John or Mary are coming, I asked you who precisely is coming!

Next, compare (25) to its minimal pair in (26), where the IDAY response seems felicitous. B's answer to A's question in (26), while non-resolving, still satisfies p-POS in nearly all possible contexts (excluding only those contexts in which it is common ground that John coming *lowers* the probability of Sue or Mary coming).[14] Therefore, if we are correct in taking IDAY to be a diagnostic for QuD-accommodation, it is unclear why the response is appropriate in this case if p-POS alone constitutes what constrains the relationship between utterances and the QuD: QuD-accommodation is not needed for B's utterance to satisfy p-POS relative to A's question, and so A has no grounds for objecting. Similarly, in (27), B's answer fully resolves A's question but adds unsolicited information, thus satisfying POS (and, trivially, p-POS) but not NEG relative

[14]We thank Benjamin Spector (p.c.) for suggesting that we pursue this line of argumentation.

to it. A would thus, again, have no grounds for objecting to it on a view that POS, but not NEG, is the constraint governing this discourse.[15]

(26) A: Did John, Sue and Mary all come to the party?
B: John did.
A: I didn't ask you whether John came, I asked you whether all of them did!

(27) A: Is John coming to the party?
B: He's coming with his pet hamster.
A: I didn't ask you whether he is bringing his pet hamster, I asked you (only) whether he is coming!

The main takeaway here is that IDAY is not merely a response that one can choose to voice whenever a question they posit is answered in a suboptimal way. If that were the case, A would have every reason to object in (25), where B's answer is suboptimal in that it is partial. Rather, IDAY seems to target those responses that violate NEG, even if these satisfy a positive constraint. We think the contrasts in (28)-(29) further illustrate this. In both cases, B's answer in the (a)-example satisfies NEG but is only partial, and A's IDAY response seems infelicitous, whereas B's answer in the (b)-example satisfies only p-POS, and A's IDAY response seems felicitous.

(28) a. A: When is John coming to the party?
B: He's coming between 10 and 10:30.
A: #I didn't ask you whether he was coming between 10 and 10:30, I asked you when precisely he is coming!

b. A: Is John coming to the party before 10:20?
B: He's coming between 10 and 10:30.
A: I didn't ask you whether he was coming between 10 and 10:30, I asked you whether he is coming before 10:20!

[15] We note, as has been pointed out to us by Kai von Fintel (p.c.), that if B utters instead the answer *Only John came*, an IDAY response seems significantly degraded. As far as we can see, nothing in our current argumentation predicts this contrast. We leave a detailed discussion of this issue for further research.

(29) **Context:** A is a detective looking for a murderer on the loose, all she knows about the killer is that her name is *Suzy*.

a. A: What is John's sister's name?
B: It's something that starts with *S*.
A: #I didn't ask you what letter it starts with, I asked you what it is!

b. A: Is John's sister called *Suzy*?
B: It's something that starts with *S*.
A: I didn't ask you what letter it starts with, I asked you whether it's *Suzy*!

Given that what distinguishes the infelicitous IDAY responses from the felicitous ones is whether they follow an answer that satisfies NEG or not, and given that IDAY seems to flag a violation of some discourse principle, it seems to us that a theory that does not posit NEG among its discourse principles would be unable to account for these patterns.

Finally, it should come as no surprise that IDAY is a natural response to the infelicitous examples in (19)-(21) above, where over-information renders an answer infelicitous. Since the reason for the infelicity is that the speaker's utterance forces the addressee to accommodate a QuD even though the speaker is not in a position to make such a demand, IDAY is all the more warranted. This is shown in (30) below.

(30) Border agent: Which countries did you visit?
Sue: I visited London and Marseille.
Border agent: I didn't ask you which cities you visited, I asked about the countries!

4.5 On the role of probabilities

Having defended NEG as a necessary constraint on discourse, let us slightly digress to comment on the relation between our arguments for NEG, and another debate regarding the discourse constraints governing how questions are to be addressed, that has taken center stage in recent years. At the heart of that debate is the question of whether such constraints should be formulated in probabilistic terms, and if so, how exactly.

As a matter of fact, that debate is usually held only relative to the positive constraint,[16] demanding that utterances provide enough information given a

[16] E.g., Büring 2003; Russell 2012; Hawkins & Goodman 2017; Feinmann 2020; Van Rooy 2004; Warstadt & Agha 2022; Warstadt et al. 2026.

question, and is primarily concerned with what constitutes enough information. According to the non-probabilistic POS (32), contributing enough information means *eliminating* an answer to the question,[17] whereas according to the probabilistic p-POS (33), it means *shifting the probability* of an answer. All sides of this debate take for granted, however, that the constraint on answers in discourse must be *stronger than* Stalnaker's (1978) informativity constraint INF (31), which demands that all utterances convey information; i.e., eliminate worlds from the context set. Given that all positive approaches demand that an answer provide information and place further constraints on the kind of information required, they all subsume Stalnakerian informativity.

(31) **Informativity (INF):** An utterance of a declarative sentence ϕ is felicitous given c only if $[\![\phi]\!] \cap c \neq c$.

(32) **Positive Question-Answer constraint (POS):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q}\rangle$ only if $\exists q \in \mathcal{Q}$: $q \cap [\![\phi]\!] = \varnothing$.

(33) **Probabilistic positive Question-Answer constraint (p-POS):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q}, P\rangle$ only if $\exists q \in \mathcal{Q} : P(q|c \cap [\![\phi]\!]) \neq P(q|c)$.

(34) **Negative Question-Answer constraint (NEG):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q}\rangle$ only if $\forall q \in \mathcal{Q}$: $q \subseteq [\![\phi]\!] \vee q \cap [\![\phi]\!] = \varnothing$.

The first thing to note here, then, is that if one finds our argument for the necessity of NEG (34) compelling, the consequentiality of this debate is immediately diminished. As is noted briefly in Section 4.2, if NEG is independently required, the choice between INF and POS becomes arbitrary, and the

[17] Another non-probabilistic positive constraint is provided by Agha & Warstadt (2020), who—to account for discourses such as (i)—propose that knowing that the information provided is *false* must eliminate an answer to the question.

(i) a. Q: Is John going to Coachella?
A: He is either going to Coachella or Lollapalooza.

b. Q: Did Mary come in first?
A: She didn't come in last.

Knowing that it not the case that John is going to Coachella or Lollapalooza—i.e., knowing that he is not going to either—resolves the question in (i-a). Knowing that it is not the case that Mary did not come in last—i.e., knowing that she did come in last—resolves the question in (i-b).

same goes for p-POS. Any utterance that conveys information by eliminating worlds from the context—thus satisfying INF—must eliminate entire cells from the partition to satisfy NEG, thus also satisfying both POS and p-POS. In other words, regardless of the formulation of our positive constraint—be it INF, POS, p-POS, or anything else that is at least as strong as INF—we end up with a system that requires an answer to eliminate a cell in the question relative to which it is uttered, while refraining from cutting through any of the non-eliminated cells.

Second, we can also explain *why* the debate over probabilities has been conducted exclusively within the confines of the positive approach: A probabilistic formulation of NEG, we will show next, is equivalent to its non-probabilistic formulation, and is therefore useless from the perspective of obtaining different empirical coverage. To show this, we must provide a probabilistic version of the negative view, and to do that, it will be useful to provide a formal definition of what it means for a constraint to be negative.

NEG essentially demands that the effect of updating the context set with any given utterance should be uniform across all cells in the partition imposed by the QuD. More specifically, it requires that for each cell, either all the worlds in it "survive" the update, or all be eliminated by the update. But there is a natural way to generalize this kind of constraint with respect to other properties—we can define a family of negative constraints in the following way:

(35) A question-answer constraint is *negative* with respect to some function f on pairs of propositions if for any declarative sentence ϕ and model of discourse containing (at least) a context c and QuD $\mathcal{Q}$, ϕ satisfies the constraint *iff*: $\forall q \in \mathcal{Q} : \forall q', q'' \subseteq q : f([\![\phi]\!], q') = f([\![\phi]\!], q'')$.

For NEG in particular, the function f required to hold uniformly of every subset of any given cell in a question $\mathcal{Q}$ is simply the function $f(p, q) = 1\ iff\ q \cap p \neq \varnothing$. Plugging this function in the definition in (35), we get a constraint which is satisfied for ϕ iff each cell in the QuD either entails its meaning (if f returns 1 for every subset of the cell) or contradicts it (if f returns 0 for every subset of the cell). This is indeed equivalent to NEG.[18]

[18] For completeness, we can define a family of positive constraints abstractly as well. Such constraints demand that updating the context with a given utterance change the status of at least one cell with respect to a certain property. In the case of POS, at least one cell should be eliminated from the context set. But again, abstracting away from the nature of the change required to occur, we can generalize as follows:

(i) A constraint on the relation between answers and questions is positive with respect to some function f on pairs of propositions if for any declarative sentence ϕ and

Is there a probabilistic constraint that is negative in the sense defined in (35), which meaningfully differs from our non-probabilistic NEG? As it turns out, the answer is no. Negative constraints demand that utterances not differentiate between worlds within the same cell in the partitioned context, with respect to some property. NEG in particular demands that utterances not eliminate worlds from a cell while leaving their other cellmate worlds untouched. If we wish to inject probability into this schema, we may demand that utterances not shift the probability of a certain subset of a cell more than it does any other subset in the same cell, as spelled out in (36). To our understanding, this is the only way to define a constraint of this sort that is both probabilistic and negative, within the semantic framework we are assuming, in which content is represented as sets of possible worlds.[19]

(36) **Probabilistic negative Question-Answer constraint (p-NEG):** An utterance of a declarative sentence ϕ is felicitous given a model $\langle c, \mathcal{Q}, P\rangle$ only if $\forall q \in \mathcal{Q}\ \forall q', q'' \subseteq q : \frac{P(q' \mid [\![\phi]\!])}{P(q')} = \frac{P(q'' \mid [\![\phi]\!])}{P(q'')}$.

But while consistent, this constraint is equivalent to, and is thus a useless notational variant of, the non-probabilistic NEG. Let us sketch out a proof. To see that p-NEG is at least as strong as NEG, consider an utterance ϕ which violates NEG; i.e. it eliminates a subset of worlds from a certain cell in $\mathcal{Q}$, while keeping the other subsets intact. The probability of the subset of eliminated worlds must be 0, while the probability of the rest of the cell's subsets is necessarily greater than 0 (otherwise they too would be eliminated). Thus, ϕ violates the p-NEG. We conclude that any utterance which satisfies p-NEG also satisfies NEG; i.e., p-NEG is at least as strong as NEG. This is not surprising once we notice that the notion of change relied on by p-NEG (the function f in (35)) is weaker than the one relied on by NEG; any utterance that eliminates a certain set of worlds also shifts its probability, but not vice versa. Since being uniform with respect to a weaker function constitutes a stronger demand, p-NEG is indeed bound to be at least as strong as NEG.

Next, we show that NEG is at least as strong as p-NEG. Consider an utterance ϕ which satisfies NEG. For any $q \in \mathcal{Q}$, this means that either $q \subseteq [\![\phi]\!]$

any model of discourse containing (at least) a context c and QuD $\mathcal{Q}$, ϕ satisfies the constraint *iff*: $\exists q \in \mathcal{Q} : f([\![\phi]\!], q) \neq f(\top, q)$ (where $\top$ denotes the tautological proposition).

POS meets this definition relative to the same f defined above for NEG, and p-POS meets the definition relative to the function $f(p, q) = P(q|p)$, where P is the model's probability parameter.

[19] But see fn. 20.

or $q \cap [\![\phi]\!] = \varnothing$. If $q \subseteq [\![\phi]\!]$, then for any $q' \subseteq q$ it holds that $q' \subseteq [\![\phi]\!]$, and from Bayes' theorem we get $P(q' \mid [\![\phi]\!]) = \frac{P(q')}{P([\![\phi]\!])}$, or equivalently $\frac{P(q'|[\![\phi]\!])}{P(q')} = \frac{1}{P([\![\phi]\!])}$. This means that $\frac{P(q'|[\![\phi]\!])}{P(q')}$ is independent of q', and so it trivially follows that $\frac{P(q'|[\![\phi]\!])}{P(q')} = \frac{P(q''|[\![\phi]\!])}{P(q'')}$ for any $q', q'' \subseteq q$, satisfying p-NEG. On the other hand, if $q \cap [\![\phi]\!] = \varnothing$ then $P(q' \mid [\![\phi]\!]) = 0$, and so $\frac{P(q'|[\![\phi]\!])}{P(q')} = 0$. Again, we see that $\frac{P(q'|[\![\phi]\!])}{P(q')}$ is independent of q', and p-NEG is likewise satisfied. Hence, NEG is at least as strong as p-NEG. Together with the previous direction, we can conclude that the two constraints are in fact equivalent, and so the introduction of probability in p-NEG does not have any empirical payoff. This explains why such an approach has never been pursued in the literature.[20]

The takeaway from this discussion is that in a system with NEG, introducing probabilities into the formulation of the constraints that determine what it means to address a question does not have any payoff; not for the positive constraints and not for the negative ones.

4.6 The reductionist approach

Before concluding, we would be remiss if we do not consider how the points in this chapter interact with an alternative approach to discourse constraints, which some take to be the "null hypothesis", which attempts to derive what appear to be constraints on utterances in discourse from domain general considerations of rational agents in discourse. To the best of our knowledge, the approach was first articulated in a passage by Carlson (1982), who proposes a game-theoretic analysis of discourse.[21] In the passage, cited almost in its entirety below, Carlson defines the notion of *relevance*, which he takes to be the property of utterances that renders them admissible in a particular discourse.

> The essential logic of relevance is best visible in a number of its goal-directed near synonyms: *important, useful, helpful.*

[20] It is interesting to note that if we adopt a more radical approach to the introduction of probability into our system and allow utterances to directly denote probabilistic information, it becomes possible to define a negative constraint which meaningfully differs from NEG. Such a radical implementation has indeed been proposed by e.g. Swanson (2006) and Moss (2018). In Appendix A, we sketch out how this system can be used to define a constraint which is weaker than NEG in that it is satisfied by a super-set of those utterances which satisfy NEG.

[21] This view is embedded in the Rational Speech Act framework, when it incorporates QuDs; e.g., Kao, Wu, Bergen, & Goodman 2014; Lassiter & Goodman 2017; Spector 2017; Degen 2023.

> Two important points emerge from these paraphrases. First, *relevant* is a relational word: a move may be relevant (useful) for one purpose but irrelevant (useless) for another. Second, relevance is relative to the *aims* of a discussion—or the aims of individual discussants if their aims diverge.
>
> Hence to determine whether a given move in a given dialogue is relevant, one has to determine whether it furthers the aims of one or more participants of that dialogue. This is the core intuition which particular examples and explanations should go back to. For instance, an answer is relevant to a question if anything is. Why? Because the aim of the questioner is to create a consensus about the (or an) answer to his question, and answering is the best way to further that aim. [...]
>
> The present definition of relevance creates a link between the concepts of relevance and rationality. A rational player is by definition one who follows his optimal strategy (or strategies)—i.e., those which (according to the theory) most effectively further his aims. This means that a rational dialogue player makes only relevant moves. This implies that the Gricean maxim of relevance becomes a corollary of the game-theoretical reconstruction of dialogue. In the dialogue game context, 'Be relevant!' says nothing more than 'Be rational!'.
>
> (Carlson 1982: 45-46)

On this approach—which takes for granted a view of discourse as rational cooperation among interlocutors—the felicity of a declarative utterance depends directly on the extent to which the information it conveys helps discourse participants promote their goals; e.g., help them answer a particular question that is being discussed, arrive at certain decisions that need to be made, or do anything, really, that they would like to achieve. There are two ways in which this view is more parsimonious than any of the approaches we have been considering so far. First, given that relevance is relativized to the goals of discourse participants, this approach does not assume a formal representation of discourse in which a QuD is explicitly represented.[22] More importantly, this approach needs not (and in fact, to our understanding, does

[22]See Hawkins, Tsvilodub, Bergey, Goodman, & Franke 2025: 2 for a discussion of the relations between overt questions in discourse and the implicit goals of interlocutors, which fits well with this approach. Greatly simplifying, they suggest that an interlocutor's choice of question provides information about their likely underlying goals.

not) posit a constraint regulating the admissibility of answers given a question, instead reducing the patterns we discuss throughout this chapter to domain-general rational considerations. Since virtually everyone accepts that rational considerations play a role in the felicity patterns of utterances in discourse, this approach promises significant reduction in the assumptions needed to explain the facts. We therefore call it the *Reductionist Approach* (RA).

The arguments in this chapter do not directly bear on many of the points of tension between RA and the approaches discussed above, pertaining to the role that constraints on question-answer pairs play in the cognitive architecture; are they domain specific, or an epiphenomenon of domain-general cognition? Rather, the point of contact between our argument for NEG and RA is as follows: Rational agents want to maximize utility while minimizing cost, so their utterances must thus be as useful as possible in advancing their goals, but not more costly than alternative utterances that are just as useful. We have demonstrated that there are instances in which over-informative answers should be ruled out, and are not ruled out due to manner considerations penalizing structurally complex sentences when a simpler alternative is available. Thus, any system with the property of *downward monotonicity*, as defined in (17), does not adequately capture the empirical landscape. From the perspective of RA this means that, either the utility function assumed in this framework should sometimes deem certain sentences less useful than sentences that they entail, or the cost function must directly penalize over-informative sentences.

4.7 Conclusion

We have argued that utterances in discourse are constrained in a way that forces them to have a uniform effect with respect to the Question under Discussion: the meaning of a declarative sentence must either be entailed by or contradict each cell in the QuD. This constraint, which we call the negative Question-Answer constraint, or NEG in short, faces an obvious challenge: It seems too strict. It often seems enough for an utterance get us closer, in some way, to resolving the QuD; a requirement that is imposed by positive constraints on discourse. While we generally do not oppose the claim that positive constraints of this sort play a role in discourse, our main argument in this chapter is that these are not enough; we must additionally posit a constraint like NEG.

We open our defense of this claim by challenging the standard methodology used to adjudicate between the competing views, which takes felicity

of an overt question-answer pair as sufficient evidence in this debate. As we show, given the availability of a QuD-accommodation mechanism, which all approaches must independently posit, the felicity of an overt pair does not by itself provide evidence for or against any particular formulation of the constraint governing how questions must be addressed. On the other hand, since QuD-accommodation can ameliorate, but never deteriorate, a discourse—infelicitous answers constitute more reliable evidence.

We then provide our positive case for NEG, which makes use of the asymmetry between the theoretical role that felicitous and infelicitous answers play in the debate. Unlike positive constraints, which do not place an upper bound on the information that an answer may provide, NEG is violated by over-informative answers. NEG is therefore better positioned to account for certain infelicitous question-answer pairs in which the answer contains too much information to satisfy NEG, but enough information to satisfy POS. We show that there are indeed such cases. And while NEG can account for them by simply assuming that they involve discourses in which QuD-accommodation is *blocked*, its positive rivals cannot do so by claiming that QuD-accommodation is *forced*.

Our second argument for NEG introduces the "*I didn't ask you*" (IDAY) test as a diagnostic for QuD-accommodation, modeled after von Fintel's (2004) "*Hey, wait a minute!*" test for presupposition accommodation. We show that speakers can legitimately resist QuD-accommodation by voicing an IDAY objection to answers that fail to satisfy NEG relative to an explicit question, even if those answers do satisfy positive constraints. Conversely, IDAY cannot be used to protest partial answers that nonetheless satisfy NEG. The systematic contrasts we provide illustrate that NEG must be among the grammatical principles constraining discourse, as weaker principles alone cannot explain the pattern of appropriate IDAY responses.

Our conclusion that the strongest formulation of the constraint on addressing questions in discourse—namely, NEG—is required is, we think, a welcome one for independent reasons. The tension between different conceptions of what it means to address a question is particularly pertinent for processes that are sensitive to formal alternatives, and many researchers rely on NEG as the deciding factor determining which formal alternatives to a given utterance are considered in the calculation of exclusive inferences; e.g., Magri (2009), Fox & Katzir (2011), Križ & Spector (2021), Bar-Lev (2021, 2023). Our independent arguments for NEG here thus strengthen the theoretical footing that this literature relies on.

Several questions remain open. The most pressing of these concerns the nature of QuD-accommodation itself. Throughout this chapter, we appeal

to this mechanism quite freely, treating it as available whenever the overt discourse fails to satisfy NEG but the addressee can plausibly reconstruct an alternative QuD relative to which it does. Yet, as our argument from over-informative answers makes clear, QuD-accommodation is not always available, and we (and the field) currently lack a theory of what governs its distribution.

Appendix: A Negative probabilistic constraint that is not a notational variant of NEG

Here we offer a way of incorporating probability distributions into our model of discourse that allows us to formulate a constraint which tracks more closely to NEG, but is nevertheless weaker than NEG itself. The key is to redefine the notion of context update, so that propositions can directly alter the shape of the probability parameter. To do so, we adopt a pragmatic model that is homomorphic to the Stalnakerian one, with probability functions replacing worlds as primitive elements. The model is, essentially, a naive implementation of a proposal by Swanson (2006) and Moss (2015a,b, 2018)—greatly simplified for presentational purposes—who suggest that sentences have sets of probability functions as their semantic value, supplanting the classical notion of content with a probabilistic one.[23] The context set c can then itself be thought of as a set of probability functions. Thus, any constraint which, in the Stalnakerian model, blocks the update of c with certain sets of *worlds* exhibiting particular properties, can be recast in the model we adopt below to block update with certain sets of *probability functions* that exhibit those same properties, *including NEG*.

To define a negative constraint in a probabilistic framework, we follow Swanson (2006); Moss (2015b) in baking probability into the semantics of declarative utterances themselves. One can do so by defining a new primitive domain, $\mathcal{D}_p$, of probability functions over worlds; i.e., measure functions from the power set of logical space, $\mathbf{P(W)}$ to real numbers in [0,1]. Such functions will play in our probabilistic semantics the same role that worlds do in classical semantics. But while any world fully determines the truth and falsity of every proposition (for any proposition, it is either in its extension or not), a probability function does not; it can assign some propositions to 1, others to 0, and yet others to values in between. These functions can thus be thought

[23] Other relevant inspirations include the insights on probability operators in Yalcin 2010 and Lassiter 2011, which nevertheless differ quite radically in their use of probability functions from the proposal we adopt here.

of as representing a possible state of the world, given our epistemic uncertainty about whether certain propositions obtain; namely, the state in which the likelihood that any proposition p obtains is $P(p)$.

This view of the role of probabilities assumes that a given set of evidence, and in particular our shared knowledge, does not deterministically decide the probability of all propositions whose truth is left open by this set; i.e., two distinct possible states can agree on which propositions are true or false, but differ in the probabilities assigned to propositions whose truth has not been determined.

With these assumptions, we can define the *p-context* as the set of all probability functions compatible with the shared knowledge of discourse participants (37). Since we assume here that the notion of meaning is inherently relativized to probability, the meanings that sentences denote are themselves probabilistic. We posit that declarative sentences denote functions from probability values—*degrees* between 0 and 1—to sets of probability functions. These functions, which we term *p-propositions*, intuitively state that the probability of a certain set of worlds is equal to its degree argument (38). In sentences that lack any overt probabilistic particles, we assume that the d-argument is saturated by an implicit argument whose value is 1. Thus, the utterance *it is raining*, denotes the set of all probability functions that map the proposition that it is raining to 1, as exemplified in (39).[24]

(37) **Probabilistic context set:** $\widetilde{c} = \{P \in \mathcal{D}_p : \mid P$ is compatible with the shared knowledge of discourse participants$\}$.

(38) **Probabilistic propositions:**

a. We call $\widetilde{p}$ a *p(robabilistic)-proposition* if it is in $\mathcal{D}_{\langle d,\langle p,t\rangle\rangle}$, where d is a probability value (a number between 0 and 1) and p is a probability function.

b. Given a classical proposition p, its probabilistic extension is the function $\widetilde{p}$ that takes a probability value d and returns the set of probability functions which assign p this value: $\widetilde{p} = \lambda d.\ \lambda P.\ P(p) = d$

[24] There is a choice point here regarding how to treat the closure of the degree argument in sentences without an overt probability operator. For simplicity, we assume here that it is done by plugging in the value 1, the result of which is that simple sentences like *it's raining* convey that it is completely certain that it's raining (similarly to the classical model). Another approach, in the spirit of Lassiter (2011), has that argument saturated by a default "positive form" operator, which maps the p-proposition to the set of probability functions which assign rain to a probability value higher than a certain threshold. On this view, the sentence *it's raining* conveys that the chance that it is raining is above some contextually-determined value. This might explain, for example, the contribution of operators like *certainly*. We refer the reader to Lassiter's work for a more detailed discussion.

(39) $[\![\text{It's raining}]\!] = \lambda P.\ P(\{w \mid \text{It's raining in } w\}) = 1$

This notion of p-proposition *subsumes* the classical notion, as it is strictly more expressive. Given a classical proposition p, its truth conditions can be expressed by feeding the value 1 to its probabilistic counterpart $\widetilde{p}$, yielding the set of probability functions that assign 1 to p. But of course, plugging in other values to $\widetilde{p}$, or quantifying over them, gives us a range of meanings that cannot be expressed in the classical system. This additional expressive power allows us to analyze probability operators like *certainly*, *probably*, *maybe*, etc. as directly quantifying over the d-argument of p-propositions (Moss, 2015b). A toy implementation is given in (40a)-(40b).[25]

(40) a.$[\![\text{Maybe it's raining}]\!] = \lambda P.\ P(\{w \mid \text{It's raining in } w\}) > 0$
b.$[\![\text{It's probably raining}]\!] = \lambda P.\ P(\{w \mid \text{It's raining in } w\}) > 0.5$

Given this semantics for propositions, we can keep to the same process of context update via intersection of the proposition with the context, as in the classical Stalnakerian model. We can also now define the QuD simply as a *p-question* that partitions the p-context, as in (41).

(41) **Probabilistic question:**

a.A *p-question* is a set of p-propositions.

b.Given a p-question $\widetilde{q}$ containing n p-propositions $\widetilde{p}_1, ..., \widetilde{p}_n$, and a p-context $\widetilde{c}$, we define the partition imposed on $\widetilde{c}$ by $\widetilde{q}$ as follows:
For any $d_1, ..., d_n$ n-tuple of probability values we define a cell:
$q_{d_1,...,d_n} = \{P \in \widetilde{c} \mid P \in \widetilde{p}_1(d_1) \ \wedge \ \ldots \ \wedge \ P \in \widetilde{p}_n(d_n)\}$
We define the partition $\widetilde{Q}$ as the set of non empty cells:
$\widetilde{Q} = \{q_{d_1,...,d_n} \mid q_{d_1,...,d_n} \neq \varnothing\}$

More specifically, we assume that questions in this probabilistic framework are sets of p-propositions. The partition a p-question imposes on $\tilde{c}$ divides it to cells such that the probability functions in each cell assign the same probability to each p-proposition in the question.[26]

[25]The definitions for the probability operators below are syncategorematic, but of course, we can isolate the semantic contribution of the operators as follows:

(i) a.$[\![\text{maybe}]\!] = \lambda\tilde{p}_{\langle d,\langle p,t\rangle\rangle}\lambda P_p.\ \exists n > \mathbf{0} : \tilde{p}(n)(P)$
b.$[\![\text{probably}]\!] = \lambda\tilde{p}_{\langle d,\langle p,t\rangle\rangle}\lambda P_p.\ \exists n > \mathbf{0.5} :\ \tilde{p}(n)(P)$

[26]This definition ignores issues of granularity in the probability values $d_1, ..., d_n$. If we allow these values to range over any real numbers between 0 and 1, the partitions we end up deriving may be very fine grained. This would mean that virtually any non-trivial

We have thus defined an embedding of the classical Stalnakerian pragmatic model within an enriched model, whose primitive objects are probability functions, rather than worlds. It should come as no surprise, then, that we can reproduce the constraints posited on addressing questions in the old model, within our modified one. Specifically, we can define both negative and positive constraints along the lines of NEG (34) and POS (32). These are given in (43)-(44), alongside a modified version of Informativity (42), for completeness.

This model does not suffer from the same issue that prevented us from defining a useful negative constraint in a probabilistic model in which utterances cannot directly alter the shape of the probability parameter; namely, that any proposition that shifts the probability of certain sets of worlds without eliminating a cell, must also rule out some worlds, thus violating NEG.

(42) **Probabilistic Informativity (p-INF):** A p-proposition $\widetilde{p}$ is *informative* relative to a p-context set $\widetilde{c}$ if $\widetilde{c} \cap \widetilde{p} \neq \widetilde{c}$

(43) **Probabilistic NEG:** A p-proposition $\widetilde{p}$ addresses p-question $\widetilde{Q}$ in a p-model $\langle \widetilde{c}, \widetilde{\mathcal{Q}} \rangle$ if $\exists \widetilde{\mathcal{Q}}' \subseteq \widetilde{\mathcal{Q}}$: $\widetilde{p} \cap \widetilde{c} = \bigcup \widetilde{\mathcal{Q}}'$.

(44) **Probabilistic POS:** A p-proposition $\widetilde{p}$ addresses p-question $\widetilde{Q}$ in a p-model $\langle \widetilde{c}, \widetilde{\mathcal{Q}} \rangle$ if $\exists \widetilde{\mathcal{Q}}' \subsetneq \widetilde{\mathcal{Q}}$: $\widetilde{p} \cap \widetilde{c} \subseteq \bigcup \widetilde{\mathcal{Q}}'$.

To see this, imagine a p-context $\widetilde{c}$, partitioned by the question *is it raining?* and assume that $\widetilde{c}$ contains no information about the chances of rain; namely, for every probability value d, there exists $P \in \widetilde{c}$ such that $P(\{w \mid \text{It's raining in } w\}) = d$. The partition we end up with is such that for each possible probability value d, it contains a cell containing all probability functions that assign d to $\{w \mid \text{It's raining in } w\}$. It is easy to see that all sentences in (45) satisfy both p-NEG and p-POS (43)-(44) relative to this partition.

(45) a.It's raining.

b.Maybe it's raining.

c.It's probably raining.

Each of these sentences states that the probability of rain is equal to, or greater than, a certain value—1 in (45a), 0 in (45b), and 0.5 in (45c)—and thus eliminates all and only those cells in which $P(\{w \mid \text{It's raining in } w\})$ returns a lower value, leaving all other cells intact. Notice that while the sentence

proposition eliminates a cell from the QuD. A discussion on whether this is a welcome result or not is beyond the scope if this chapter, but we note that a natural way to raise the bar of what constitutes eliminating a cell is to define some granularity on the values of $d_1, ..., d_n$ that would restrict their density.

in (45a) satisfies NEG in the classical model as well, (45b) and (45c) do not. This demonstrates that p-NEG is a slightly weaker notion than NEG.

Bibliography

Agha, Omar & Paloma Jeretič. 2022. Weak necessity modals as homogeneous pluralities of worlds. In *Semantics and Linguistic Theory (SALT)*, 831–851.

Agha, Omar & Alex Warstadt. 2020. Non-resolving responses to polar questions: A revision to the QUD theory of relevance. In *Proceedings of Sinn und Bedeutung*, volume 24, 17–34.

Bar-Lev, Moshe E. 2021. An implicature account of homogeneity and non-maximality. *Linguistics and Philosophy* 44. 1045–1097.

Bar-Lev, Moshe E. 2023. Obligatory implicatures and the relevance of contradictions. *Manuscript, TAU* .

Barcan, Ruth C. 1946. A functional calculus of first order based on strict implication. *The journal of symbolic logic* 11. 1–16.

Barwise, Jon & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4. 159–219.

Bassi, Itai & Ezer Rasin. 2018. Equational-intensional relative clauses with syntactic reconstruction. In *Sinn und Bedeutung (SuB)*, volume 22, 143–159.

Beaver, David I & Brady Z Clark. 2009. *Sense and sensitivity: How focus determines meaning*. John Wiley & Sons.

Beck, Sigrid & Uli Sauerland. 2000. Cumulation is needed. *Natural language semantics* 8. 349–371.

Beghelli, Filippo. 1993. A minimalist approach to quantifier scope. In *Proceedings of NELS 23*, volume 1, 65–80. Amherst, MA: GSLI, University of Massachusetts.

Benbaji, Ido. 2021. Restricting the fourth reading. *Semantics and Linguistic Theory (SALT)* 31. 221–240.

Benbaji, Ido & Omri Doron. 2023. Adversative only is only only. In *Proceedings of Sinn und Bedeutung*, volume 27, 64–80.

Benbaji-Elhadad, Ido. 2023. Preliminaries for a substitution theory of de re. In *SALT*, 358–378.

Benbaji-Elhadad, Ido. 2026a. A challenge to unified accounts of non-canonical belief reports. In *Proceedings of NELS 56*. To appear.

Benbaji-Elhadad, Ido. 2026b. Reconstruction, bare quantification, and the fourth reading. Manuscript, MIT.

Bennett, Michael. 1977. A guide to the logic of tense and aspect in english. *Logique et analyse* 20. 491–517.

van Benthem, Johan. 1977. Tense logic and standard logic. *Logique et Analyse* 20. 437–450.

Bhatt, Rajesh. 2002. The raising analysis of relative clauses: Evidence from adjectival modification. *Natural Language Semantics* 10. 43–90.

Bhatt, Rajesh & Yael Sharvit. 2005. A note on intensional superlatives. In *Semantics and Linguistic Theory (SALT)*, 62–79.

Blumberg, Kyle & Harvey Lederman. 2021. Revisionist reporting. *Philosophical Studies* 178. 755–783.

Brame, Michael. 1968. A new analysis of the relative clause: Evidence for an interpretive theory. Unpublished manuscript, MIT.

Büring, Daniel. 2003. On d-trees, beans, and b-accents. *Linguistics and philosophy* 26. 511–545.

Burnett, Heather. 2016. *Gradability in natural language: Logical and grammatical foundations*, volume 7. Oxford University Press.

Carlson, Lauri. 1982. *Dialogue games: An approach to discourse analysis*. Dordrecht, Holland: D. Reidel Publishing Company.

Carnap, Rudolf. 1950. *Logical foundations of probability*. Chicago, IL: University of Chicago Press.

Cecchetto, Carlo & Caterina Donati. 2015. *(re)labeling*. MIT Press.

Champollion, Lucas. 2020. Distributivity, collectivity and cumulativity. In *Wiley's companion to semantics*. Wiley.

Champollion, Lucas. 2025. Truthmaker semantics and natural language semantics. *Language and Linguistics Compass* 19. e70004.

Charnavel, Isabelle. 2019. *Locality and logophoricity: A theory of exempt anaphora*. Oxford University Press.

Charnavel, Isabelle. 2022. The latest take on intensional superlatives. In *Proceedings of Sinn und Bedeutung 26*, ed. Daniel Gutzmann & Sophie Repp, 206–224. University of Cologne.

Charnavel, Isabelle. 2024. The superlative clause hypothesis. In *Proceedings of the 40th West Coast Conference on Formal Linguistics*, ed. Jiayi Lu, Erika Petersen, Anissa Zaitsu, & Boris Harizanov, 42–51. Somerville, MA: Cascadilla Proceedings Project.

Charnavel, Isabelle. 2026. Appealing to superlative clauses. *Natural Language & Linguistic Theory* 44. 1–57.

Charnavel, Isabelle & Dominique Sportiche. 2016. Anaphor binding: What french inanimate anaphors show. *Linguistic Inquiry* 47. 35–87.

Collingwood, Robin George. 1940/2001. *An essay on metaphysics*. Oxford University Press.

Cresswell, Maxwell John. 1990/2012. *Entities and indices*, volume 41. Springer Science & Business Media.

Degen, Judith. 2023. The rational speech act framework. *Annual Review of Linguistics* 9. 519–540.

Demirok, Ömer. 2019. Scope theory revisited: lessons from pied-piping in wh-questions. Doctoral Dissertation, Massachusetts Institute of Technology.

den Dikken, Marcel, Richard Larson & Peter Ludlow. 2018. Intensional transitive verbs and abstract clausal complementation. In *Non-propositional intentionality*, 46–94. Oxford University Press.

van der Does, J. M. 1992. Applied quantifier logics: Collectives, naked infinitives. Doctoral Dissertation, University of Amsterdam, Amsterdam. PhD dissertation.

Elliott, Patrick D. 2023. A flexible scope theory of intensionality. *Linguistics and Philosophy* 46. 333–378.

Enç, Mürvet. 1982. Tense without scope: An analysis of nouns as indexicals. Doctoral Dissertation, University of Wisconsin, Madison.

Enç, Mürvet. 1986. Towards a referential analysis of temporal expressions. *Linguistics and Philosophy* 9. 405–426.

Farkas, Donka F & Kim B Bruce. 2010. On reacting to assertions and polar questions. *Journal of semantics* 27. 81–118.

Fauconnier, Gilles. 1984. *Mental spaces*. Cambridge, MA: MIT Press.

Feinmann, Diego. 2020. Lexical illusions,(non-) maximality, and invisible gaps. Doctoral Dissertation, University of Sheffield.

Feinmann, Diego. 2026. A note on relevance. *Journal of Semantics* 1–15.

Ferreira, Marcelo. 2005. Event quantification and plurality. PhD dissertation, Massachusetts Institute of Technology, Cambridge, MA.

Fine, Kit. 2017. Truthmaker semantics. In *A companion to the philosophy of language*, chapter 22, 556–577. John Wiley & Sons, Ltd.

von Fintel, Kai. 2004. Would you believe it? the king of france is back! (presuppositions and truth-value intuitions). In *Descriptions and beyond*. Oxford: Oxford University Press.

von Fintel, Kai. 2008. What is presupposition accommodation, again? *Philosophical perspectives* 22. 137–170.

von Fintel, Kai, Danny Fox & Sabine Iatridou. 2014. Definiteness as maximal informativeness. In *The art and craft of semantics: A festschrift for irene heim*, 70, 165–174. Cambridge, MA: MITWPL.

von Fintel, Kai & Irene Heim. 2011. *Intensional semantics*. MIT lecture notes.

Fodor, Janet. 1970. The linguistic description of opaque contexts. Doctoral Dissertation, MIT.

Forbes, Graeme. 2000. Objectual attitudes. *Linguistics and Philosophy* 23. 141–183.

Forbes, Graeme. 2006. *Attitude problems: An essay on linguistic intensionality*. Oxford University Press.

Fox, Danny. 2003. On logical form. In *Minimalist syntax*, 82–123. Malden, MA: Blackwell.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural language semantics* 19. 87–107.

Francez, Itamar. 2018. Summative existentials. *Linguistic Inquiry* 49. 723–739.

Gajewski, Jon Robert. 2005. Neg-raising: Polarity and presupposition. Doctoral Dissertation, Massachusetts Institute of Technology.

Gallin, Daniel. 1975. *Intensional and higher-order modal logic*. Amsterdam: North-Holland.

Green, Mitchell S. 1995. Quantity, volubility, and some varieties of discourse. *Linguistics and Philosophy* 18. 83–112.

Grice, Herbert P. 1975. Logic and conversation. In *Speech acts*, 41–58. New York: Academic Press.

Groenendijk, Jeroen & Martin Stokhof. 1982. Semantic analysis of wh-complements. *Linguistics and Philosophy* 5. 175–233.

Groenendijk, Jeroen Antonius Gerardus & Martin Johan Bastiaan Stokhof. 1984. Studies on the semantics of questions and the pragmatics of answers. Doctoral Dissertation, Univ. Amsterdam.

Grosu, Alexander & Manfred Krifka. 2007. The gifted mathematician that you claim to be: Equational intensional 'reconstruction' relatives. *Linguistics and Philosophy* 30. 445–485.

Grosu, Alexander & Manfred Krifka. 2022. A note on the gifted mathematician that you claim to be. *Linguistics and Philosophy* 45. 1227–1233.

Haslinger, Nina. 2023. Pragmatic constraints on imprecision and homogeneity. Doctoral Dissertation, University of Göttingen.

Hawkins, Robert D, Polina Tsvilodub, Claire Augusta Bergey, Noah D Goodman & Michael Franke. 2025. Relevant answers to polar questions. *Philosophical Transactions of the Royal Society B: Biological Sciences* 380.

Hawkins, Robert X. D. & Noah D. Goodman. 2017. Why do you ask? the informational dynamics of questions and answers. *PsyArXiv* .

Heim, Irene. 1987. Where does the definiteness restriction apply: Evidence from the definiteness of variables. In *The representation of (in)definiteness*, 21–42. Cambridge, MA: MIT Press.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of semantics* 9. 183–221.

Heim, Irene. 2018. Functional readings without type-shifted noun phrases. In *Reconstruction effects in relative clauses*, 283–302. Berlin: De Gruyter.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

Heycock, Caroline. 2005. On the interaction of adjectival modifiers and relative clauses. *Natural Language Semantics* 13. 359–382.

Heycock, Caroline. 2019. Relative reconstructions. In *Reconstruction effects in relative clauses*, 87–112. Berlin/Boston: De Gruyter.

Hintikka, Jaakko. 1962. *Knowledge and belief: An introduction to the logic of the two notions*. Ithaca, NY: Cornell University Press.

Horn, Laurence R. 1984. Toward a new taxonomy for pragmatic inference: Q-based and r-based implicature. In *Meaning, form, and use in context: Linguistic applications*, 11–42. Washington, DC: Georgetown University Press.

Hulsey, Sarah & Uli Sauerland. 2006. Sorting out relative clauses. *Natural Language Semantics* 14. 111–137.

Hyska, Megan Alexandra. 2015. Discourse-level information structure and the challenge of metadiscursives. Master's report, The University of Texas at Austin, Austin, TX.

Ionin, Tania & Ora Matushansky. 2002. Dps with a twist: A unified analysis of russian comitatives. In *Formal approaches to slavic linguistics 11: The amherst meeting 2002*. Ann Arbor, MI: Michigan Slavic Publications.

Ioup, Georgette. 1975. Some universals for quantifier scope. In *Syntax and semantics volume 4*, 37–58. Brill.

Jacobson, Pauline. 2016. The short answer: Implications for direct compositionality (and vice versa). *Language* 92. 331–375.

Kamp, Hans. 1971. Formal properties of 'now'. *Theoria* 37. 227–273.

Kao, Justine T, Jean Y Wu, Leon Bergen & Noah D Goodman. 2014. Nonliteral understanding of number words. *Proceedings of the National Academy of Sciences* 111. 12002–12007.

Kayne, Richard S. 1994. *The antisymmetry of syntax*. MIT press.

Kennedy, Christopher. 2012. Adjectives. In *Routledge companion to philosophy of language*, 328–341. New York: Routledge.

Keshet, Ezra. 2008. Good intensions: Paving two roads to a theory of the de re/de dicto distinction. Doctoral Dissertation, MIT.

Keshet, Ezra. 2010a. Situation economy. *Natural Language Semantics* 18. 385–434.

Keshet, Ezra. 2010b. Split intensionality: A new scope theory of de re and de dicto. *Linguistics and Philosophy* 33. 251–283.

Keshet, Ezra & Florian Schwarz. 2019. De re/de dicto. In *The oxford handbook of reference*, 167–202. Oxford: Oxford University Press.

Kratzer, Angelika. 1977. What 'must' and 'can' must and can mean. *Linguistics and Philosophy* 1. 337–355. Publisher: Springer Netherlands.

Kratzer, Angelika. 1981. The notional category of modality. *Formal semantics: The essential readings* 289–323.

Kratzer, Angelika. 2003. The event argument and the semantics of verbs. Unpublished manuscript. UMass Amherst.

Kratzer, Angelika. 2007/2023. Situations in Natural Language Semantics. In *The Stanford encyclopedia of philosophy*. Metaphysics Research Lab, Stanford University.

Kratzer, Angelika. 2008. On the plurality of verbs. In *Event structures in linguistic form and interpretation*, volume 5 of *Language, Context and Cognition*, 269–300. Berlin: Mouton de Gruyter.

Krifka, Manfred. 1986. Nominalreferenz und zeitkonstitution: Zur semantik von massentermen, pluraltermen und aspektklassen. Doctoral Dissertation, University of Munich.

Krifka, Manfred. 1992. Thematic relations as links between nominal reference and temporal constitution. In *Lexical matters*, 29–53. Stanford: CSLI Publications.

Križ, Manuel. 2015. Aspects of homogeneity in the semantics of natural language. Doctoral Dissertation, University of Vienna.

Križ, Manuel. 2019. Homogeneity effects in natural language semantics. *Language and linguistics compass* 13. e12350.

Križ, Manuel & Mora Maldonado. 2018. An experimental note on distributivity and scope. Manuscript, LSCP.

Križ, Manuel & Benjamin Spector. 2021. Interpreting plural predication: Homogeneity and non-maximality. *Linguistics and Philosophy* 44. 1131–1178.

Ladusaw, William. 1977. Some problems with tense in PTQ. In *Texas Linguistic Forum*, volume 6, 90–102. University of Texas.

Ladusaw, William. 1979. Polarity sensitivity as inherent scope relations. Doctoral Dissertation, The University of Texas at Austin.

Landman, Fred. 1996. Plurality. In *The handbook of contemporary semantic theory*, 425–457. Oxford: Blackwell.

Landman, Fred. 2000. *Events and plurality*. Dordrecht: Kluwer Academic Publishers.

Larson, Richard K. 1998. Events and modification in nominals. In *Semantics and Linguistic Theory (SALT)*, ed. Devon Strolovitch & Aaron Lawson, 145–168. Ithaca, NY: Cornell University.

Larson, Richard K. 2002. The grammar of intensionality. In *Logical form and language*, 228–262. Oxford: Clarendon Press.

Lassiter, Daniel. 2011. *Measurement and modality: The scalar basis of modal semantics*. Ph. D. thesis, New York University.

Lassiter, Daniel & Noah D Goodman. 2017. Adjectival vagueness in a bayesian model of interpretation. *Synthese* 194. 3801–3836.

Levinson, Stephen C. 2000. *Presumptive meanings: The theory of generalized conversational implicature*. Cambridge, MA: MIT Press.

Lewis, David. 1979. Scorekeeping in a language game. *Journal of philosophical logic* 8. 339–359.

Lewis, David. 1988a. Relevant implication. *Theoria* 54. 161–174.

Lewis, David. 1988b. Statements partly about observation. *Philosophical papers* 17. 1–31.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice theoretical approach. In *Meaning, use, and interpretation of language/walter de gruyter*, 303–323. Berlin: de Gruyter.

Lumsden, Michael. 1988. *Existential sentences: Their structure and meaning*. Routledge.

Magri, Giorgio. 2009. A theory of individual-level predicates based on blind mandatory scalar implicatures. *Natural language semantics* 17. 245–297.

Marty, Paul. 2019. A note on non-distributive belief ascription. *Snippets* 36. 1–2.

Mayr, Clemens & Viola Schmitt. 2024. Non-de dicto construals as a unified phenomenon. In *SALT 34*.

Mayr, Clemens & Viola Schmitt. 2026a. Generalized non-de dicto: Underspecification and discourse stability. Unpublished Manuscript.

Mayr, Clemens & Viola Schmitt. 2026b. Generalized non-de dicto: Underspecification and discourse stability. Unpublished ms.

McNally, Louise. 1992. An interpretation for the english existential construction. Doctoral Dissertation, University of California, Santa Cruz.

Meisenbichler, Paul. 2025. Interactions of worlds, times and locations: The limits of (non-local) index shifting. In *Proceedings of the 55th Annual Meeting of the North East Linguistic Society (NELS 55)*, 137–147.

Meisenbichler, Paul. 2026. Upper limit effects in dps and the interdependence of worlds and times. In *Proceedings of Sinn und Bedeutung*, volume 30, 762–778.

Merchant, Jason. 2005. Fragments and ellipsis. *Linguistics and philosophy* 27. 661–738.

Milsark, Gary L. 1974. Existential sentences in english. Doctoral Dissertation, MIT.

Mirrazi, Zahra. 2024. Indefinites in negated intensional contexts: An argument for world-skolemized choice functions. *Semantics and Pragmatics* 17. 6–1.

Moltmann, Friederike. 2019. Intensional relative clauses and the semantics of variable objects. In *Reconstruction effects in relative clauses*, 427–454. Berlin, Boston: De Gruyter.

Moltmann, Friederike. 2020a. Truthmaker semantics for natural language: Attitude verbs, modals, and intensional transitive verbs. *Theoretical linguistics* 46. 159–200.

Moltmann, Friederike. 2020b. Variable objects and truthmaking. In *Metaphysics, meaning, and modality: Themes from kit fine*. Oxford University Press.

Montague, Richard. 1970. Universal grammar. *Theoria* 36. 373–398.

Montague, Richard. 1973. The proper treatment of quantification in ordinary English. In *Approaches to natural language*, 221–242. Dordrecht: Reidel.

Moss, Sarah. 2015a. Credal dilemmas. *Noûs* 49. 665–683.

Moss, Sarah. 2015b. On the semantics and pragmatics of epistemic vocabulary. *Semantics and pragmatics* 8. 5–1.

Moss, Sarah. 2018. *Probabilistic knowledge*. Oxford University Press.

Musan, Renate. 1995. On the temporal interpretation of noun phrases. Doctoral Dissertation, MIT.

Musan, Renate. 1999. Temporal interpretation and information-status of noun phrases. *Linguistics and philosophy* 621–661.

Nouwen, Rick. 2022. Unknown numbers. In *Measurements, numerals and scales: Essays in honour of stephanie solt*, 231–246. Cham: Springer International Publishing.

O'Leary, Maura C. 2022. About time: Lexical, structural, and discourse constraints on the temporal interpretation of nominal predicates. Doctoral Dissertation, UCLA.

Partee, Barbara. 1975. Montague grammar and transformational grammar. *Linguistic Inquiry* 6. 203–300.

Partee, Barbara H. 1995. Lexical semantics and compositionality. In *An invitation to cognitive science, volume 1: Language*, 311–360. Cambridge, MA: MIT Press.

Partee, Barbara Hall. 1973. Some structural analogies between tenses and pronouns in english. *The Journal of Philosophy* 70. 601–609.

Pasternak, Robert. 2018. Thinking alone and thinking together. In *SALT*, 546–565.

Percus, Orin. 2000. Constraints on some other variables in syntax. *Natural Language Semantics* 8. 173–229.

Percus, Orin. 2020. Index-dependence and embedding. *The Wiley Blackwell Companion to Semantics.* 1–31.

Reinhart, Tanya. 1997. Quantifier scope: How labor is divided between qr and choice functions. *Linguistics and philosophy* 335–397.

Roberts, Craige. 1996/2012. Information structure: Towards an integrated formal theory of pragmatics. *Semantics and pragmatics* 5. 6–1.

Russell, Benjamin. 2012. Probabilistic reasoning and the computation of scalar implicatures. *Unpublished doctoral dissertation, Brown University* .

Ruys, Eddy G. 1992. The scope of indefinites. Doctoral Dissertation, Utrecht University. PhD dissertation.

Salzmann, Martin. 2017. *Reconstruction and resumption in indirect a'-dependencies: On the syntax of prolepsis and relativization in (swiss) german and beyond*, volume 117 of *Studies in Generative Grammar*. Berlin/Boston: De Gruyter.

Santorio, Paolo. 2013. Descriptions as variables. *Philosophical Studies* 164. 41–59.

Scha, Remko. 1981. Distributive, collective and cumulative quantification. In *Formal methods in the study of language*, vol. 2, 483–512. Amsterdam: Mathematisch Centrum.

Schachter, Paul. 1973. Focus and relativization. *Language* 19–46.

Schein, Barry. 1993. *Plurals and events*. Cambridge, MA: MIT Press.

Schlenker, Philippe. 2004. Conditionals as definite descriptions. *Research on language and computation* 2. 417–462.

Schlenker, Philippe. 2006. Ontological symmetry in language: A brief manifesto. *Mind & language* 21. 504–539.

Schmitt, Viola. 2019. Pluralities across categories and plural projection. *Semantics and Pragmatics* 12. 17–1.

Schmitt, Viola. 2020. Cumulation across attitudes and plural projection. *Journal of Semantics* 37. 557–609.

Schmitt, Viola. 2023. Are there pluralities of worlds? *Journal of Semantics* 40. 153–178.

Schwager, Magdalena. 2009. Speaking of qualities. *Semantics and Linguistic Theory (SALT)* 19. 395–412.

Schwarz, Florian. 2021. Intensional transitive verbs. In *The wiley blackwell companion to semantics*. John Wiley & Sons, Inc.

Seldin, Jonathan P. 2006. The logic of curry and church. In *Handbook of the history of logic*, volume 5, 819–873. Elsevier.

Shanon, Benny. 1976. On the two kinds of presuppositions in natural language. *Foundations of language* 14. 247–249.

Sharvit, Yael. 1999. Functional relative clauses. *Linguistics and Philosophy* 447–478.

Sharvit, Yael. 2007. Two reconstruction puzzles. In *Direct compositionality*, chapter 10, 336–360. Oxford University Press.

Sichel, Ivy. 2014. Resumptive pronouns and competition. *Linguistic Inquiry* 45. 655–693.

Simons, Mandy, Judith Tonhauser, David Beaver & Craige Roberts. 2010. What projects and why. In *Semantics and Linguistic Theory (SALT)*, 309–327.

Spector, Benjamin. 2017. The pragmatics of plural predication: Homogeneity and non-maximality within the rational speech act model. In *Proceedings of the Amsterdam Colloquium*, 435–444.

Sperber, Dan & Deirdre Wilson. 1986. *Relevance: Communication and cognition*. Oxford: Blackwell.

Stalnaker, Robert. 1977. Pragmatic presuppositions. In *Proceedings of the Texas conference on performatives, presuppositions, and implicatures. Arlington, VA: Center for Applied Linguistics*, 135–148. ERIC.

Stalnaker, Robert C. 1978. Assertion. In *Pragmatics*, 315–332. Brill.

Steedman, Mark. 1987. Combinatory grammars and parasitic gaps. *Natural Language & Linguistic Theory* 5. 403–439.

Steedman, Mark. 1988. Combinators and grammars. In *Categorial grammars and natural language structures*, 417–442. Dordrecht: D. Reidel.

Steedman, Mark. 2011. *Taking scope: The natural semantics of quantifiers*. Cambridge, MA: MIT Press.

Sternefeld, Wolfgang. 1998. Reciprocity and cumulative predication. *Natural language semantics* 6. 303–337.

Swanson, Eric Peter. 2006. Interactions with context. Doctoral Dissertation, Massachusetts Institute of Technology.

Szabó, Zoltán Gendler. 2006. Comments on kai von fintel's paper. Unpublished manuscript, revised version of comments given at ohio workshop on presupposition accommodation.

Szabó, Zoltán Gendler. 2010. Specific, yet opaque. *Amsterdam Colloquium* 17. 32–41.

Szabó, Zoltán Gendler. 2011. Bare quantifiers. *Philosophical Review* 120. 247–283.

Szabolcsi, Anna. 1989. Bound variables in syntax (are there any?). In *Semantics and contextual expression*, 294–318. Dordrecht: Foris.

Tichý, Pavel. 1971. An approach to intensional analysis. *Noûs* 5. 273–297.

Tonhauser, Judith. 2002. A dynamic semantic account of the temporal interpretation of noun phrases. In *SALT*, 286–305.

Van Rooy, Robert. 2004. Utility, informativity and protocols. *Journal of philosophical logic* 33. 389–419.

Vergnaud, Jean-Roger. 1974. French relative clauses. Doctoral Dissertation, MIT.

Warstadt, Alex & Omar Agha. 2022. Testing bayesian measures of relevance in discourse. In *Proceedings of Sinn und Bedeutung*, volume 26, 865–886.

Warstadt, Alex, Omar Agha & Michael Franke. 2026. Quantifying epistemic relevance. *Open Mind* 10. 502–546.

Wehbe, Jad. 2023. Against the lexical view of cumulative inferences. In *Proceedings of Sinn und Bedeutung*, volume 27, 693–711.

Wehbe, Jad. 2026. Plural predication and scope. Doctoral dissertation, Massachusetts Institute of Technology, Cambridge, MA.

Williamson, Timothy. 2013. *Modal logic as metaphysics*. Oxford University Press.

Winter, Yoad. 1997. Choice functions and the scopal semantics of indefinites. *Linguistics and philosophy* 399–467.

Winter, Yoad. 2000. Distributivity and dependency. *Natural language semantics* 8. 27–69.

Yablo, Stephen. 2014. *Aboutness*. Princeton: Princeton University Press.

Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5. 916–937.

Yanovich, Igor. 2015. Expressive power of "now" and "then" operators. *Journal of Logic, Language and Information* 24. 65–93.

Zimmermann, Thomas Ede. 1989. Intensional logic and two-sorted type theory. *Journal of Symbolic Logic* 54. 65–77.

Zimmermann, Thomas Ede. 2006. Monotonicity in opaque verbs. *Linguistics and Philosophy* 29. 715–761.

Zimmermann, Thomas Ede. 2018. Fregean compositionality. In *The science of meaning: Essays on the metatheory of natural language semantics*. Oxford University Press.